1. Let X_t be a continuous time Markov chain with states $0, 1, 2, 3$ and jump rates

$$q(0,1) = 5, \ q(0,2) = q(0,3) = 3, \ q(1,2) = q(1,3) = q(2,3) = 1, \ q(3,2) = 4.$$

   (a) Draw a picture. Which states are recurrent?

   (b) Find $P_0(T_1 < \infty)$ and $P_0(T_2 < \infty)$.

   (c) Starting in state 0 what is the total expected amount of time spent in state 1?

   (d) Starting in state 2 what is the probability of being in state 3 at time $t = 10$?

   (e) Starting in state 0 what is the long term fraction of time we spend in state 3?

2. (a) Uranium has a half-life of 4.5×10^9 years. This means the time T it takes a uranium atom to decay into a lead atom is exponentially distributed with some rate λ, such that $P(T > 4.5 \times 10^9) = \frac{1}{2}$. Find λ.

   (b) A gram of uranium contains $X_0 = 2.5 \times 10^{21}$ atoms. Each atom decays into lead independently at rate λ. Let X_t be the number of uranium atoms remaining after t years. Convince yourself that X_t is a continuous time Markov chain. Find its jump rates.