Math 6720: Probability II

This is the second semester of graduate probability. Topics include Markov chains, probability on trees and networks, Brownian motion, martingales in continuous time, stationary sequences and ergodic theory.

1 Approximate List of Topics By Date

Reading the relevant reference before we cover the topic in class is an excellent way to stay on top of the material.

week	due	topic	reference
Jan 24,26		Reversible Markov chains, electrical networks	LPW 9, LP 2
Jan 29,31,2		Loop-erased random walk, uniform spanning tree, Wilson's algorithm	LP 4
Feb 5,7,9		Ising model, Gibbs measures, Metropolis algorithm	LPW 3
Feb 12,14,16	ps 1	Markov chain mixing, coupling	LPW 4,5
Feb 19		***** Winter Break *****	
Feb 21,23		Branching processes	LP 5, D
Feb 26,28,2	ps 2	Random graphs	AS
Mar 5,7,9		Gaussian processes, construction of Brownian motion	MP 1
Mar 12,14,16	ps 3	Path properties of Brownian motion	MP 1
Mar 19,21,23		Continuous time martingales, optional stopping	MP 2
Mar 26,28,30	ps 4	Skorohod embedding, Donsker's theorem, Martingale CLT	MP 5,
Apr 2,4,6		***** Spring Break *****	
Apr 9,11,13		Gaussian free field	
Apr 16,18,20		Stationary sequences, measure-preserving maps	D 6
Apr 23,25,27	ps 5	Birkhoff ergodic theorem, subadditive ergodic theorems	D 6
Apr 30,2,4		*****Student Presentations*****	
May 7,9		*****Student Presentations*****	

The exact topics and schedule are subject to change!

2 Textbooks, with links to the electronic versions if available

[AS] Alon,Spencer, The Probabilistic Method, 3rd edition.
[D] Durrett, Probability: Theory & Examples, 4th edition.
[LP] Lyons,Peres, Probability on trees and networks.
[LPW] Levin,Peres,Wilmer, Markov chains and mixing times, 2nd edition.
[MP] Morters,Peres, Brownian Motion.
[W] Williams, Probability with Martingales.

3 Problem Sets

Aim for the writing style of a research paper. Many scientfic papers end with "acknowledgements" where you can thank X & Y for inspiring conversations, Z for pointing out a gap in your proof, W for feeding and encouraging you when the going got tough, and Gimme Coffee for providing an essential raw material.

The vast majority of math papers are typeset using LaTeX.

4 Group work policy

Working in groups is strongly encouraged! Discuss all you want, then write the solution in your own words. (Hint: It will be hard for your classmate to write the solution in their own words if you give them your written solution to look at.) You are free to use any online or offline resource for the problem sets, provided you clearly state when you've done so. If someone gave you a good idea, it would be polite to thank them.

5 Presentations

Present in a group of 2–5 on a probability research topic of your choice. If you need ideas for a topic, browse the textbooks, especially Lyons and Peres. You can choose between two fora:

- Present in class during the last two weeks of the semester; or
- Present in the arXiv seminar, which meets Tuesdays 11:40-12:55.

The length of the presentation is 50 minutes if you present in class, or up to 75 minutes if you present in the arXiv seminar. Your group can choose how to split up the time among the group members. In that time the group should aim to state one theorem, place it in context (why is it interesting? what gap in human knowledge does it fill? what related things are true/false/easy/hard/known/unknown?), and convey the main idea of the proof.

Expect a lot of questions from me and your classmates that will slow you down! If a practice presentation takes 30 minutes with no questions, then you're in good shape.

6 Grading

The problem sets count for 70% of the grade. The presentation counts for 30%. The presentation grade is based both on your specific piece of the presentation, and on the quality and coherence of your group's presentation as a whole.