Quiz 2 Solution
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1. Find the solution set.

$$\begin{array}{rcrcrcrcl} 4x & + & y & + & 2z & - & w & = & 0 \\ 8x & - & 2y & + & z & & & = & 0 \\ -4x & & & - & z & + & 3w & = & 0 \end{array}$$

The associated matrix is

$$\begin{pmatrix} 4 & 1 & 2 & -1 \\ 8 & -2 & 1 & 0 \\ -4 & 0 & -1 & 3 \end{pmatrix}.$$

There's no need to write the last column since it will never contain anything but zeros. Subtract twice the first row from the second row, and add the first row to the third row:

$$\begin{pmatrix} 4 & 1 & 2 & -1 \\ 0 & -4 & -3 & 2 \\ 0 & 1 & 1 & 2 \end{pmatrix}.$$

Now add $\frac{1}{4}$ times the second row to the third row:

$$\begin{pmatrix} 4 & 1 & 2 & -1 \\ 0 & -4 & -3 & 2 \\ 0 & 0 & \frac{1}{4} & \frac{5}{2} \end{pmatrix}.$$

There's no pivot in the last column, so $w$ is a free variable: $w = t$, where $t$ is any real number. Now by back substition

$$\frac{1}{4}z + \frac{5}{2}w = 0 \Rightarrow z = -10w = -10t;$$

$$-4y - 3z + 2w = 0 \Rightarrow y = \frac{1}{4}(2w - 3z) = \frac{1}{4}(2t + 30t) = 8t;$$

$$4x + y + 2z - w = 0 \Rightarrow x = \frac{1}{4}(w - y - 2z) = \frac{1}{4}(t - 8t + 20t) = \frac{13t}{4}.$$

So the solution set is

$$SS = \{(\frac{13t}{4}, 8t, -10t, t) \mid t \in \mathbb{R}\}.$$

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