

Quiz 3 Solution
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1. Find the solution set.

$$\begin{array}{ccccccccc} x & - & 2y & & & + & w & = & 2 \\ & & & & & -z & + & 3w & = & 0 \\ 2x & - & 4y & + & 3z & - & 7w & = & 4 \end{array}$$

The associated matrix is

$$\begin{pmatrix} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & -1 & 3 & 0 \\ 2 & -4 & 3 & -7 & 4 \end{pmatrix}.$$

Subtract twice the first row from the last row:

$$\begin{pmatrix} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 3 & -9 & 0 \end{pmatrix}.$$

Now add three times the second row to the last row:

$$\begin{pmatrix} 1 & -2 & 0 & 1 & 2 \\ 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

This matrix is in row echelon form. The second and fourth columns lack pivots, so y and w are free variables: $y = t_1$, $w = t_2$, where t_1 and t_2 may be any real numbers. By back substitution,

$$-z + 3w = 0 \Rightarrow z = 3w = 3t_2;$$

$$x - 2y + w = 2 \Rightarrow x = 2y - w + 2 = 2t_1 - t_2 + 2.$$

Thus the solution set is

$$SS = \{(2t_1 - t_2 + 2, t_1, 3t_2, t_2) \mid t_1, t_2 \in \mathbb{R}\}.$$