Quiz 4 Solution
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1. Determine whether the matrix

$$A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 2 & 5 \\ 0 & -3 & 1 \end{pmatrix}$$

has an inverse, and find the inverse if it exists.

Form the augmented matrix

$$\left(\begin{array}{ccc|ccc} 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 5 & 0 & 1 & 0 \\ 0 & -3 & 1 & 0 & 0 & 1 \end{array} \right).$$

Swap the first and second rows:

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 5 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 & 0 & 1 \end{array} \right).$$

Subtract three times the second row from the third row:

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 5 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array} \right).$$

The left side is now in row-echelon form. Since the bottom row is not zero, the matrix has an inverse. To find it, subtract 5 times the bottom row from the top row:

$$\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & 15 & 1 & -5 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array} \right).$$

Add twice the second row to the top row:

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 17 & 1 & -5 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array} \right).$$

Finally, multiply the second row by -1:

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 17 & 1 & -5 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -3 & 0 & 1 \end{array} \right).$$

The matrix on the right side is A^{-1}.

To check, we compute

$$AA^{-1} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 2 & 5 \\ 0 & -3 & 1 \end{pmatrix} \begin{pmatrix} 17 & 1 & -5 \\ -1 & 0 & 0 \\ -3 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

and

$$A^{-1}A = \begin{pmatrix} 17 & 1 & -5 \\ -1 & 0 & 0 \\ -3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 2 & 5 \\ 0 & -3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$