Quiz 5 Solution
GSI: Lionel Levine
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1. Let $A = \begin{pmatrix} 1 & -2 \\ 0 & 2 \end{pmatrix}$.

   (a) Find A^{-1}.

   (b) Express A^{-1} as a product of elementary matrices.

   (c) Express A as a product of elementary matrices.

(a) Form the augmented matrix

$$\left(\begin{array}{cc|cc} 1 & -2 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{array}\right).$$

Add the bottom row to the top row, using the elementary matrix $E_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$:

$$\left(\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 1 \end{array}\right).$$

Now multiply the bottom row by $\frac{1}{2}$, using the elementary matrix $E_2 = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$:

$$\left(\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & \frac{1}{2} \end{array}\right).$$

Thus $A^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & \frac{1}{2} \end{pmatrix}$.

(b) The above proceedure expressed the identity matrix as $I = E_2 E_1 A$. Therefore

$$A^{-1} = E_2 E_1 = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

(c) Taking the inverse of both sides (remember to reverse the order!) we get

$$A = E_1^{-1} E_2^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}.$$