

Quiz 8 Solution
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2/13/04

1. Determine if the vectors are linearly independent. (Please justify your answers!)

(a) $(1, 2)$, $(3, 4)$, and $(5, 6)$.

(b) $(1, 2, 1)$, $(3, 0, 2)$ and $(-1, 4, 0)$.

(a) A set of more than n vectors in $\mathbb{R}^n$ is always linearly dependent, so these three vectors in $\mathbb{R}^2$ are linearly dependent.

(b) The vectors are linearly dependent if and only if the equation

$$a(1, 2, 1) + b(3, 0, 2) + c(-1, 4, 0) = (0, 0, 0)$$

has a nontrivial solution (a, b, c) , i.e. a solution where a , b and c are not all equal to zero. This equation is equivalent to the system

$$\begin{array}{rrrrrr} a & + & 3b & - & c & = & 0 \\ 2a & & & + & 4c & = & 0 \\ a & + & 2b & & & = & 0. \end{array}$$

Row-reduce the associated matrix (remember, there's no need to write the answer column when all the answers are zero):

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \\ 1 & 2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3 & -1 \\ 0 & -6 & 6 \\ 0 & -1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 3 & -1 \\ 0 & -6 & 6 \\ 0 & 0 & 0 \end{pmatrix}.$$

There's no pivot in the last column, so c is a free variable, and the system has infinitely many solutions. Therefore the vectors are linearly dependent.