

Quiz 9 Solution
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1. (a) Find a basis for the subspace W of $\mathbb{R}^4$ given by

$$W = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_1 - x_2 + x_3 - x_4 = 0\}.$$

- (b) What is $\dim W$?

- (a) Solving for x_4

$$x_4 = x_1 - x_2 + x_3$$

we see that any vector $\mathbf{w}$ in W has the form

$$\mathbf{w} = (x_1, x_2, x_3, x_1 - x_2 + x_3)$$

for real numbers x_1 , x_2 and x_3 . Write this as a sum of three vectors

$$\begin{aligned}\mathbf{w} &= (x_1, 0, 0, x_1) + (0, x_2, 0, -x_2) + (0, 0, x_3, x_3) \\ &= x_1(1, 0, 0, 1) + x_2(0, 1, 0, -1) + x_3(0, 0, 1, 1).\end{aligned}$$

This tells us that the vectors

$$\mathbf{v}_1 = (1, 0, 0, 1), \quad \mathbf{v}_2 = (0, 1, 0, -1), \quad \mathbf{v}_3 = (0, 0, 1, 1)$$

span the space W . It is easy to check that $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are linearly independent, and hence they form a basis for W .

- (b) Since there are three basis vectors, $\dim W = 3$.

Note: A vector space has many different bases. Therefore there are many different correct answers to part (a)! For example, if you solved for x_1 instead of x_4 in the beginning, you would find a different basis.