Quiz 10 Solution
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1. Find the coordinates of the vector $(3, 2, 2)$ with respect to the basis

$$v_1 = (1, 0, 0) \quad v_2 = (1, 0, 1), \quad v_3 = (1, 1, 2).$$

We are looking for numbers a_1, a_2, a_3 such that

$$\begin{aligned} (3, 2, 2) &= a_1(1, 0, 0) + a_2(1, 0, 1) + a_3(1, 1, 2) \\ &= (a_1 + a_2 + a_3, a_3, a_2 + 2a_3). \end{aligned}$$

This gives the system of equations

$$\begin{array}{rcrcrcl} a_1 & + & a_2 & + & a_3 & = & 3 \\ & & & & a_3 & = & 2 \\ & & a_2 & + & 2a_3 & = & 2. \end{array}$$

Solving the system gives cooridnates

$$a_1 = 3, \quad a_2 = -2, \quad a_3 = 2.$$