Quiz 11 Solution
GSI: Lionel Levine
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1. Find the equation of the line that best fits the points

$$(0,0), \quad (2,0), \quad (0,1)$$

in the least-squares sense.

We are looking for an *approximate* solution to the system

$$0m + b = 0$$

$$2m + b = 0$$

$$0m + b = 1.$$

In matrix form, this reads:

$$\begin{pmatrix} 0 & 1 \\ 2 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

According to Theorem 4.35 we should find the *exact* solution to the system

$$\begin{pmatrix} 0 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} 0 & 2 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

By multiplying out the matrices, this reduces to the system

$$\begin{pmatrix} 4 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

which yields the solution $b = \frac{1}{2}$, $m = -\frac{1}{4}$. Therefore the equation of the best-fit line is

$$y = -\frac{1}{4}x + \frac{1}{2}.$$