Quiz 12 Solution
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1. Find the coordinates of the vector $\mathbf{w} = (1, 3, 5, 7)$ in the orthogonal basis

$$\begin{aligned}\mathbf{v}_1 &= (1, 1, 1, 1)\\ \mathbf{v}_2 &= (1, 1, -1, -1)\\ \mathbf{v}_3 &= (1, -1, 1, -1)\\ \mathbf{v}_4 &= (1, -1, -1, 1)\end{aligned}$$

for $\mathbb{R}^4$.

Since the basis is orthogonal, we can find the coordinates by taking dot products:

$$\begin{aligned}\mathbf{v}_1 \cdot \mathbf{w} &= 16\\ \mathbf{v}_2 \cdot \mathbf{w} &= -8\\ \mathbf{v}_3 \cdot \mathbf{w} &= -4,\\ \mathbf{v}_4 \cdot \mathbf{w} &= 0.\end{aligned}$$

This tells us that

$$\mathbf{w} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + c_4\mathbf{v}_4,$$

where

$$\begin{aligned}c_1 &= \tfrac{\mathbf{v}_1\cdot\mathbf{w}}{\mathbf{v}_1\cdot\mathbf{v}_1} = \tfrac{16}{4} = 4\\ c_2 &= \tfrac{\mathbf{v}_2\cdot\mathbf{w}}{\mathbf{v}_2\cdot\mathbf{v}_2} = \tfrac{-8}{4} = -2\\ c_3 &= \tfrac{\mathbf{v}_3\cdot\mathbf{w}}{\mathbf{v}_3\cdot\mathbf{v}_3} = \tfrac{-4}{4} = 1\\ c_4 &= \tfrac{\mathbf{v}_4\cdot\mathbf{w}}{\mathbf{v}_4\cdot\mathbf{v}_4} = \tfrac{0}{4} = 0.\end{aligned}$$

Thus the coordinates are 4, -2, -1 and 0. To check our answer, we can just compute

$$(1, 3, 5, 7) = 4(1, 1, 1, 1) - 2(1, 1, -1, -1) - (1, -1, 1, -1).$$