Quiz 14 Solution GSI: Lionel Levine 3/14/05

1. Find all real eigenvalues of the matrix

$$A = \left(\begin{array}{rrr} 4 & 0 & 0\\ 0 & 1 & 1\\ 0 & 2 & 2 \end{array}\right).$$

The matrix is in block diagonal form with blocks B = (4) and $C = \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}$. To find the eigenvalues of C, compute

$$\det \begin{pmatrix} 1-t & 1\\ 2 & 2-t \end{pmatrix} = (1-t)(2-t) - 2 = t^2 - 3t = t(t-3).$$

This tells us that the eigenvalues of C are 0 and 3, so the eigenvalues of A are 0, 3 and 4.