

Quiz 15 Solution
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3/30/05

1. Determine if the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

is diagonalizable, and if it is, find an invertible matrix S and a diagonal matrix D such that $A = SDS^{-1}$.

The characteristic polynomial of A is

$$\det \begin{pmatrix} t-1 & -1 & -1 \\ 0 & t & 0 \\ 0 & 0 & t \end{pmatrix} = t^2(t-1),$$

so the eigenvalues are 0 and 1. The eigenspaces are

$$W_0 = NS \begin{pmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$

$$W_1 = NS \begin{pmatrix} 0 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

A basis for W_0 is $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$; and a basis for W_1 is $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. So A is diagonalizable and

$$S = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$