Quiz 16 Solution
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1. Let $A = \begin{pmatrix} -2 & -8 \\ 2 & 6 \end{pmatrix}$. Find an invertible matrix S and a matrix B in Jordan-normal form such that $A = SBS^{-1}$.

The characteristic polynomial is

$$\det \begin{pmatrix} -2-t & -8 \\ 2 & 6-t \end{pmatrix} = (-2-t)(6-t) + 16 = t^2 - 4t + 4 = (t-2)^2.$$

Since there is a double root and A is not already diagonal, A is not diagonalizable. This tells us that

$$B = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}.$$

To find S, we compute the matrix

$$N = A - 2I = \begin{pmatrix} -4 & -8 \\ 2 & 4 \end{pmatrix}.$$

Since the last column of N is not zero, we let $v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and

$$v_1 = Nv_2 = \begin{pmatrix} -8 \\ 4 \end{pmatrix}.$$

The columns of S are given by v_1 and v_2:

$$S = \begin{pmatrix} -8 & 0 \\ 4 & 1 \end{pmatrix}.$$