Quiz 20 Solution
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1. Let $A$ be a $2 \times 2$ matrix with eigenvalues $\lambda_1$ and $\lambda_2$. Match each condition on the matrix $A$ with the corresponding direction field for the system $\mathbf{x}' = A\mathbf{x}$.

   (a) $\mathrm{Tr}(A)^2 < 4\det(A)$.

   (b) $\mathrm{Tr}(A)^2 = 4\det(A)$.

   (c) $\mathrm{Tr}(A) < 0$, $\det(A) > 0$, and $\lambda_1, \lambda_2 \in \mathbb{R}$.

   (d) $\lambda_1 > 0$, $\lambda_2 < 0$.

1. This has one negative eigenvalue (arrows pointing toward the origin) and one positive eigenvalue (away from the origin), so it corresponds to (d).

2. This has no real eigenvalues, so the characteristic polynomial $\lambda^2 - \mathrm{Tr}(A)\lambda + \det(A)$ must have complex roots. Therefore $b^2 - 4ac = \mathrm{Tr}(A)^2 - 4\det(A) < 0$, so this corresponds to (a).

3. This has only one eigenvalue, so the characteristic polynomial must have a repeated root. Therefore $b^2 - 4ac = 0$ and this corresponds to (b).

4. This has two real, negative eigenvalues, so $\mathrm{Tr}(A) = \lambda_1 + \lambda_2 < 0$, and $\det(A) = \lambda_1\lambda_2 > 0$. So this corresponds to (c)