

Quiz 21 Solution
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Find the Fourier series for the function

$$f(x) = \begin{cases} 0, & -2 \leq x < -1 \\ 1, & -1 \leq x < 1 \\ 0, & 1 \leq x < 2 \end{cases}, \quad f(x+4) = f(x).$$

This is an even function, so there are no sine terms. We have

$$a_0 = \frac{1}{2} \int_{-2}^2 f(x) dx = 1$$

and

$$\begin{aligned} a_n &= \frac{1}{2} \int_{-2}^2 f(x) \cos \frac{n\pi x}{2} dx \\ &= \int_0^2 f(x) \cos \frac{n\pi x}{2} dx \\ &= \int_0^1 \cos \frac{n\pi x}{2} dx \\ &= \frac{2}{n\pi} \sin \frac{n\pi}{2} \\ &= \begin{cases} -\frac{2}{n\pi} (-1)^k, & \text{if } n = 2k-1 \text{ is odd} \\ 0, & \text{if } n \text{ is even.} \end{cases} \end{aligned}$$

Therefore the Fourier series is

$$f(x) = \frac{1}{2} - \sum_{k=1}^{\infty} (-1)^k \frac{2}{(2k-1)\pi} \cos \frac{(2k-1)\pi x}{2}.$$