Math 54 Spring 2005

Solutions to Homework Section 4.4

March 9, 2005

Apply G-S to the following collections of vectors.

18. $\{\mathbf{v}_1 = (1,-1), \mathbf{v}_2 = (2,1)\}$. G-S yields $\{\mathbf{u}_1 = \frac{1}{\sqrt{2}}(1,-1), \mathbf{u}_2 = \frac{1}{\sqrt{2}}(1,1)\}$.

19. $\{\mathbf{v}_1 = (1,1,1), \mathbf{v}_2 = (1,2,2), \mathbf{v}_3 = (1,0,1)\}$. G-S yields $\{\mathbf{u}_1 = \frac{1}{\sqrt{3}}(1,1,1), \mathbf{u}_2 = \frac{1}{\sqrt{6}}(-2,1,1), \mathbf{u}_3 = \frac{1}{\sqrt{2}}(0,-1,1)\}$.

23. *Apply the Gram-Schmidt process to* $f_1 = 1$, $f_2 = x$, $f_3 = x^2$ *with* $f \cdot g = \int_{-1}^{1} fg$.

$$\mathbf{w}_1 = f_1 = 1$$

$$\mathbf{w}_2 = f_2 - \frac{f_2 \cdot \mathbf{w}_1}{\mathbf{w}_1 \cdot \mathbf{w}_1}\mathbf{w}_1 = x - 0\mathbf{w}_1 = x$$

$$\mathbf{w}_3 = f_3 - \frac{f_3 \cdot \mathbf{w}_1}{\mathbf{w}_1 \cdot \mathbf{w}_1}\mathbf{w}_1 - \frac{f_3 \cdot \mathbf{w}_2}{\mathbf{w}_2 \cdot \mathbf{w}_2}\mathbf{w}_2 = x^2 - \frac{1}{3}1 - 0\mathbf{w}_2 = x^2 - \frac{1}{3}$$

$$\mathbf{u}_1 = \mathbf{w}_1/\sqrt{2} \qquad \mathbf{u}_2 = \mathbf{w}_2/\sqrt{\frac{2}{3}} \qquad \mathbf{u}_3 = w_3/\sqrt{\frac{8}{45}} = 3\sqrt{10}/4\mathbf{w}_3$$

24. *Apply the Gram-Schmidt process to* $1-x$, $1+x$, $1+x+x^2$ *with* $f \cdot g = \int_{-1}^{1} fg$.

$$\mathbf{w}_1 = 1 - x$$

$$\mathbf{w}_2 = 1 + x - 1/2(1-x) = 3x/2 + 1/2$$

$$\mathbf{w}_3 = 1 + x + x^2 - (3/4)\mathbf{w}_1 - (-1/6)\mathbf{w}_2 = (1/6) + 2x + x^2$$

$$\mathbf{u}_1 = \mathbf{w}_1/\sqrt{(8/3)} \qquad \mathbf{u}_2 = \mathbf{w}_2/\sqrt{2} \qquad \mathbf{u}_3 = \mathbf{w}_3/\sqrt{(301/180)}$$

25. $\{\mathbf{A}_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}, \mathbf{A}_3 = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\}$. G-S is as follows:

$$\mathbf{W}_1 = \mathbf{A}_1$$

$$\mathbf{W}_2 = \mathbf{A}_2 - (\mathbf{A}_2 \cdot \mathbf{W}_1)/(\mathbf{W}_1 \cdot \mathbf{W}_1)\mathbf{W}_1 = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\mathbf{W}_3 = \mathbf{A}_3 - (\mathbf{A}_3 \cdot \mathbf{W}_1)/(\mathbf{W}_1 \cdot \mathbf{W}_1)\mathbf{W}_1 - (\mathbf{A}_3 \cdot \mathbf{W}_2)/(\mathbf{W}_3 \cdot \mathbf{W}_3)\mathbf{W}_3 = \begin{bmatrix} 1 & 4 \\ -2 & -3 \end{bmatrix}$$

$$\mathbf{U}_1 = 1/2\mathbf{W}_1 \qquad \mathbf{U}_2 = 1/\sqrt{6}\mathbf{W}_2 \qquad \mathbf{U}_3 = 1/\sqrt{30}\mathbf{W}_3$$