Math 54 Spring 2005

Solutions to Homework Section 5.1

March 11, 2005

5. $\det\begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix} = 1 \cdot 3 + 2 \cdot 2 = 7.$ INVERTIBLE.

6. $\det\begin{bmatrix} 7 & -11 \\ 2 & -4 \end{bmatrix} = -6.$ INVERTIBLE.

7. $\det\begin{bmatrix} -2 & 3 \\ 4 & -6 \end{bmatrix} = 0.$ NOT INVERTIBLE

9. $\det\begin{bmatrix} 3 & -1 \\ 4 & 0 \end{bmatrix} = 4.$ INVERTIBLE.

13. $\det\begin{bmatrix} 2 & 5 & 1 \\ 3 & -3 & 2 \\ 7 & 7 & 4 \end{bmatrix} = 2(-26) - 5(-2) + 42 = 0.$ NOT INVERTIBLE.

14. $\det\begin{bmatrix} 3 & -1 & -2 \\ -2 & 2 & 1 \\ -1 & 3 & 0 \end{bmatrix} = -2(-4) - 1(8) = 0.$ NOT INVERTIBLE.

17 Expand along the second row to get:

$$\begin{aligned} \det\begin{bmatrix} 4 & 1 & 2 & 1 \\ 3 & 0 & 0 & 0 \\ -1 & 0 & 2 & 1 \\ 4 & 5 & 0 & 2 \end{bmatrix} &= -3\det\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 1 \\ 5 & 0 & 2 \end{bmatrix} \\ &= -3(2(2-5) - 1(-10)) = -3(4) = -12 \end{aligned}$$

INVERTIBLE.

20 *Find the determinant of*

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Expanding along the first column, we get:

$$\det(\mathbf{A}) = (-1)^{1+3} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

Expanding this determinant along the first column, we get:

$$\begin{aligned}
\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} &= (-1)^2 \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \\
&= (-1)^2 \cdot \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \\
&= -1
\end{aligned}$$

25. $\det \mathbf{A} = 3x + 3 - 2x = x + 3$. The matrix is singular when $x = -3$.

26. $\det \mathbf{A} = 4x^2 - 4x - 5x = (4x - 9)x$. The matrix is singular when $x = 0$ or $x = 9/4$.

32. *Find* $\mathbf{A}$ *and* $\mathbf{B}$ *with* $\det(\mathbf{A} + \mathbf{B}) \neq \det \mathbf{A} + \det \mathbf{B}$

    Let $\mathbf{A} = \mathbf{I}_2$, $\mathbf{B} = -\mathbf{I}_2$. Then $\det(\mathbf{A} + \mathbf{B}) = \det \overrightarrow{0} = 0$ but $\det \mathbf{A} + \det \mathbf{B} = 1 + 1 = 1$.