Math 54 Worksheet 3
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1. Compute the product

$$\begin{pmatrix} 1 & 2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4 & 1 & 1 & 1 \\ -1 & -2 & 0 & 0 \\ 3 & -1 & 4 & 0 \\ 1 & 2 & 3 & -1 \end{pmatrix}.$$

   Now without solving the system

$$\begin{array}{rcrcrcrcl} 4x & + & y & + & z & + & w & = & 0 \\ -x & - & 2y & & & & & = & 0 \\ 3x & - & y & + & 4z & & & = & 0 \\ x & + & 2y & + & 3z & - & w & = & 0 \end{array},$$

   determine how many solutions it has (zero, one or infinitely many).

2. Suppose $A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$ is a 3×3 matrix. For each of the operations below, try to find a 3×3 matrix B such that multiplying A by B accomplishes the operation.

   (a) Multiplying the whole matrix by 2, so it becomes $\begin{pmatrix} 2a & 2b & 2c \\ 2d & 2e & 2f \\ 2g & 2h & 2i \end{pmatrix}$.

   (b) Multiplying the first row by 1, the second row by 2, and the third row by 3, so it becomes $\begin{pmatrix} a & b & c \\ 2d & 2e & 2f \\ 3g & 3h & 3i \end{pmatrix}$.

   (c) Multiplying the first column by 1, the second column by 2, and the third column by 3.

   (d) Swapping the second and third rows.

   (e) Swapping the second and third columns.

   (f) Adding the first row to the third row, so it becomes $\begin{pmatrix} a & b & c \\ d & e & f \\ a+g & b+h & c+i \end{pmatrix}$.

   (g) Shifting the rows downward, so it becomes $\begin{pmatrix} 0 & 0 & 0 \\ a & b & c \\ d & e & f \end{pmatrix}$.