Math 54 Worksheet 8
GSI: Lionel Levine
2/11/04

1. Determine if the vectors are linearly independent.
   (a) $(1,2)$ and $(1,3)$.
   (b) $(1,2)$, $(1,3)$ and $(1,4)$.
   (c) $(1,2)$ and $(0,0)$.
   (d) $(1,1,0)$ and $(0,1,1)$.
   (e) $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and $(5,10,20)$.

2. Find the null space. (Compare problem 1!)
   (a) $\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$
   (b) $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \end{pmatrix}$
   (c) $\begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}$
   (d) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$
   (e) $\begin{pmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 20 \end{pmatrix}$

3. Recall that a matrix A is called *nonsingular* if the only solution to the equation $A\mathbf{x} = \mathbf{0}$ is $\mathbf{x} = \mathbf{0}$; otherwise, A is called singular. For each of the conditions below, decide whether A is definitely singular, definitely nonsingular, or could be either.
   (a) $NS(A) = \{\mathbf{0}\}$.
   (b) The columns of A are linearly dependent.
   (c) There are two different vectors $\mathbf{x}$ and $\mathbf{y}$ such that $A\mathbf{x} = A\mathbf{y}$.
   (d) A is the zero matrix.
   (e) A is 3×4.
   (f) A is 4×3.
   (g) A is invertible.

Bonus Problem: A 2×2 matrix M is called a *commutator* if it can be expressed in the form $M = AB - BA$ for some 2×2 matrices A and B. Describe the span of all commutators in the space M_{22} of all 2×2 matrices.