Math 54 Worksheet 16
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1. Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$.
   (a) For which values of t (if any) is the matrix
   $$\begin{pmatrix} 1-t & 0 \\ 0 & 2-t \end{pmatrix}$$
   singular?
   (b) For which values of t (if any) is there an eigenvector v of A with eigenvalue t?
   (c) Can you find the eigenvectors associated to the eigenvalues from part (b)?
2. Let $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & -1 \end{pmatrix}$.
   (a) For which values of t (if any) is the matrix
   $$\begin{pmatrix} 1-t & 1 & 0 \\ 0 & -t & 0 \\ 0 & -1 & -1-t \end{pmatrix}$$
   singular?
   (b) For which values of t (if any) is there an eigenvector v of A with eigenvalue t?
   (c) Can you find the eigenvectors associated to the eigenvalues from part (b)?
3. Prove that if A and B are invertible $n \times n$ matrices, then AB is invertible.
4. Prove that if A is a singular $n \times n$ matrix, then AB is singular for any $n \times n$ matrix B.