Math 54 Worksheet 18
GSI: Lionel Levine
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1. (a) Write down five different 2×2 matrices A satisfying $A^2 = I$ (It might help to think about this geometrically.)
   (b) Prove that if v is an eigenvector of a matrix A with eigenvalue λ, then v is also an eigenvector of A^2, with eigenvalue λ^2.
   (c) What are the eigenvalues of the identity matrix I?
   (d) If $A^2 = I$, what can you say about the eigenvalues of A?

2. Let D be the linear transformation on $C[0, 1]$ defined by

$$D(f) = f'.$$

   (a) Find an eigenvector of D with eigenvalue 1.
   (b) What is the dimension of the eigenspace W_1?
   (c) For which real numbers λ does D have an eigenvector with eigenvalue λ?

3. Let D be the linear transformation on $C[0, 1]$ defined by

$$D(f) = f''.$$

   (a) Find an eigenvector of D with eigenvalue -1.
   (b) What is the dimension of the eigenspace W_{-1}?
   (c) Find an eigenvector of D with eigenvalue 1.
   (d) What is the dimension of the eigenspace W_1?