

Math 54 Worksheet 20
GSI: Lionel Levine
4/1/05

1. (a) Find the 2×2 matrix A corresponding to the linear transformation of projecting onto the line $y = 3x$.
(b) What is A^2 ? Is there a geometric reason for this?
(c) Suppose P is an $n \times n$ matrix corresponding to the linear transformation of projecting onto a subspace $W \subset \mathbb{R}^n$. What are the possible eigenvalues of P ?
2. An $n \times n$ matrix Q is called *orthogonal* if its columns are orthonormal.
 - (a) Prove that if Q is orthogonal, then Q is invertible and $A^{-1} = A^T$.
 - (b) Prove that if Q is orthogonal, then

$$(Qv, Qw) = (v, w)$$

for any two vectors v and w in $\mathbb{R}^n$.