Math 54 Worksheet 22 GSI: Lionel Levine 4/13/05

1. Consider the system of differential equations

$$f'(t) = f(t) - 2g(t); 
 g'(t) = -2f(t) + 4g(t).
 (1)$$

- (a) Find a second-order differential equation involving only f.
- (b) Do the same for g.
- (c) Solve these two equations for f and g separately. (remember to find the general solution!)
- (d) Does every possible choice of f and g from part (c) give you a solution to the system (1)? Can there be any extra solutions to (1) other than the ones from part (c)? What is the general solution for (1)?
- (e) Now find the eigenvalues and eigenvectors of the matrix

$$\left(\begin{array}{rrr}1 & -2\\ -2 & 4\end{array}\right).$$

Do you see a pattern?

2. Express the second-order differential equation

$$f'' - f' + 6f = 0$$

as a system of two first-order equations.