What is dimension?

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The goal of this activity is to introduce you to the notion of dimension. The movie Flatland is also a great way to learn about dimension, I recommend it.

Dimension can be thought of as ways in which you can move in a space (or a world). For now, let's think that the **dimension** of a space counts in how many independent directions you can move in the space.

1. Suppose you live in a world in which you can only move in one direction. Can you imagine such a world? Can you draw it?

2. In a 1-dimensional space you can go backward as well, does this change your drawing?

3. Suppose now you can move (forward and backward) in 2 directions. Draw this world.

4. How many dimensions does our world have? Why?

5. Are there any 0-dimensional spaces? If you think there are none, explain why. If you think there are some, describe them.

A more mathematical version of dimension

An investigation

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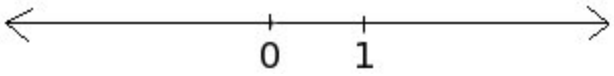
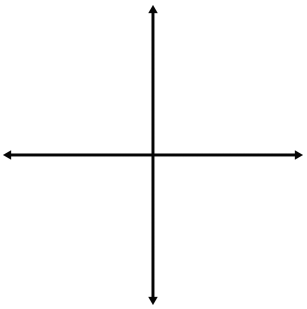
The previous handout gives a nice way to think about dimension, however it is not very precise. Mathematicians think about dimension in the following way:

Definition: The dimension of a space is how many numbers I need to specify a point in the space.

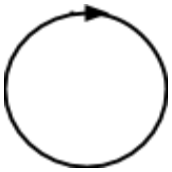
For example in a 1-dimensional space, you only need one number to specify a point. Therefore, a point corresponds to a number.



1. Why do we need 2 numbers to specify a point in the space below?



2. What about in a 3-dimensional space, why do we need here 3 numbers to specify a point?
3. What is the dimension of a hollow circle?



4. Consider now the example of the hexagon, what is the dimension of the hollow hexagon? What is the dimension of the filled hexagon?

Imagination allows us to think using 4, 5, 6, etc. numbers to define a point. However, think doesn't mean visualize; 4-dimensional spaces are very difficult to think about for experienced mathematicians. We will see some examples of this throughout the course. Just as a partial answer, we can think of time as being an independent dimension.

5. Suppose the vertical line in a 2-dimensional space represents time passing on a 1-dimensional space. Draw in the 2-dimensional plane, how Frankenstein looks like as he walks forward.

Similarly, we can think of time passing in a 2-dimensional space as the up direction in a 3-dimensional space. Then we actually live in a 4-dimensional space because we experience time. Notice a curious fact, even though we can move forward and backward, we cannot go back in time.