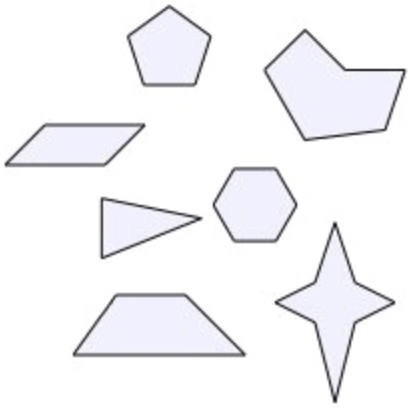
Polygons

An investigation by Laura Escobar
Math Explorer's Club

In this worksheet we will learn about different ways mathematicians think about polygons. We just saw some examples of polygons, let me include some more examples.



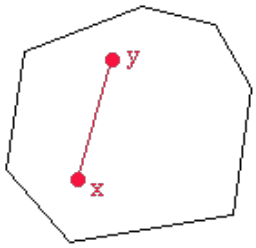
1. Try to classify the polygons on the left by stating the differences and similarities some may have.

Notice that polygons have certain pieces: points, lines and regions. We will later see a nice way to obtain such pieces. There are many ways to construct polygons, but in this course we will only consider polygons constructed in the following way.

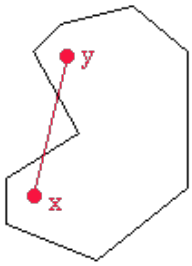
2. **Construction:** Draw a circle and 7 points on the circle. For each point you drew, connect it with the 2 closest points with a line segment. What do you get?

3. Not all of the polygons we have encountered can be constructed like this. Which examples of the polygons can be constructed in this way?

The polygons we can obtain in these way are **convex**, this means that if we take two points on the polygon and connect them using a line segment we do not go out of the polygon.



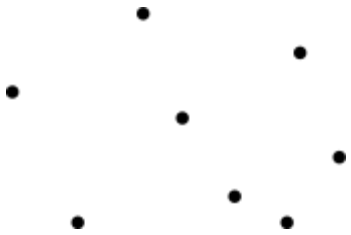
A convex polygon



A non–convex polygon

Let us now learn the way some mathematicians learn about polygons.

4. We start with finitely many points in the plane, drawn below.

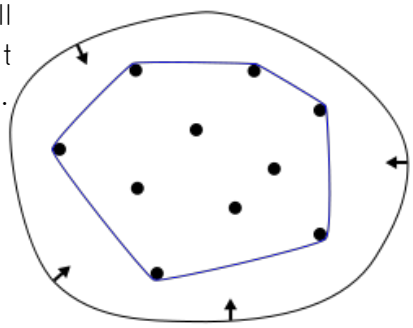


   a. First draw all the line segments connecting any two points.
   b. Now draw all the line segments connecting any two points on the lines you drew in part a.
   c. Repeat a similar process, but now drawing all line segments connecting any two points of the drawing obtained after part b. Do you get anything different?

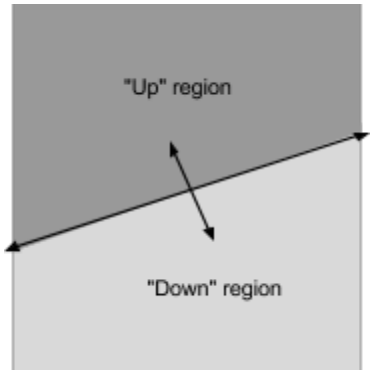
   We then obtain an object consisting of all the lines you drew, what is it?

This process you just made is called obtaining the **convex hull** of the points you drew. You can do this process for any points you draw on the plane. The outermost

5. Explain why the object you obtain is a convex polygon.
6. Suppose you hammer some nails into a wall and then take a rubber band and wrap it around the nails as in the drawing on the right. Convince yourself that this construction is the same as the one you completed in part 4.

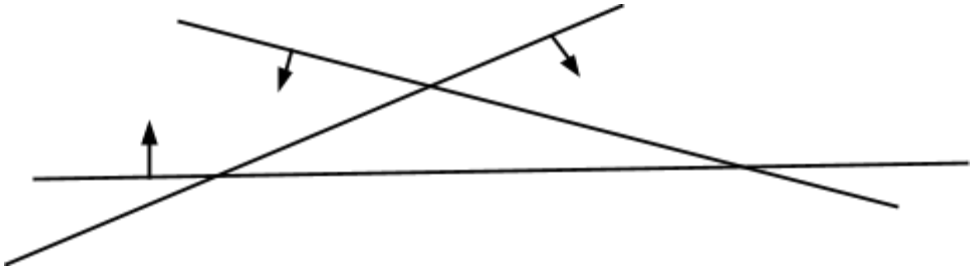


Polygons can be constructed using points or lines. We will now see how to construct a polytope using lines. A line we draw on the plane splits it into two regions, just like cutting a sandwich gives you two pieces.

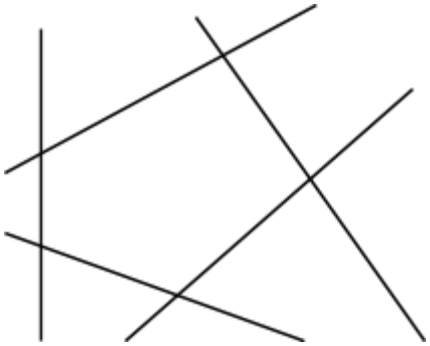


To construct polygons, we will draw lines and pick a favorite side for each line. The polygon will then be the region that is on all our favorite sides.

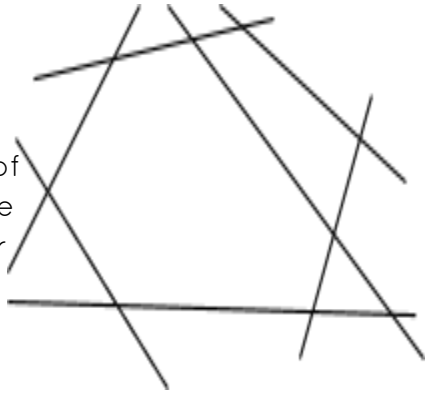
7. Consider the lines below. The little arrow indicate which is our favorite side. What polygon do we get?



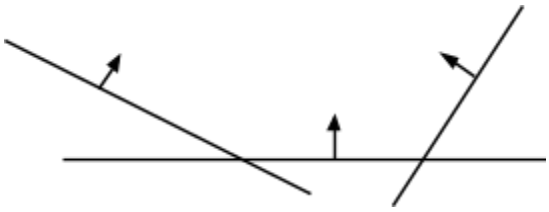
8. Now consider the following arrangement of lines. Pick the sides so that we get a pentagon.



9. Now pick the sides of the following arrangement of lines so that we get a quadrilateral. Repeat the process so that we get a hexagon. What other polygons can we get?



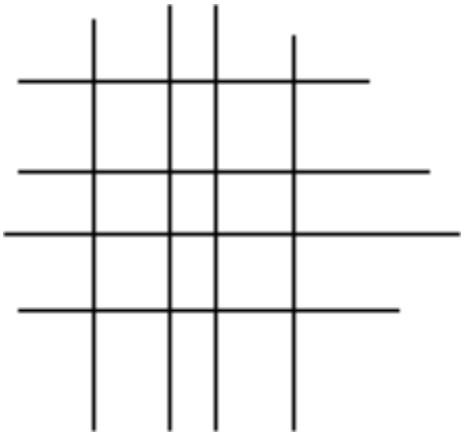
10. Consider the following arrangement of lines and regions. This one is different from the examples we have seen, how? (remember that the lines go forever in each direction.



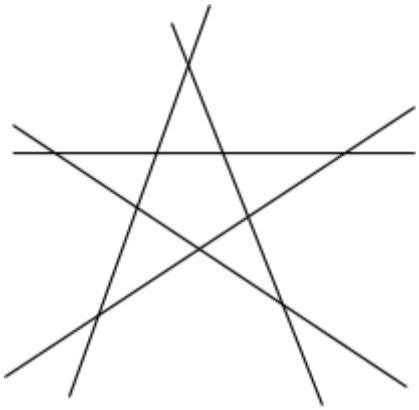
The difference between 10 and what we encountered before is that it goes in one direction forever, we call this property **unbounded**. In this course we will only work with **bounded** polygons. You can always draw a circle around a bounded polygon, this allows us to recognize them. However, an easier way to recognize bounded polygons is by walking along the perimeter of the polygon: if you loop around, then the polygon is bounded.

The process of drawing lines and picking favorite sides only gives us convex objects.

11. Consider the arrangement of lines below and suppose it gives a polygon that looks like a cross. Explain why it is impossible.



12. Consider the arrangement of lines below and suppose it gives the star polygon. Explain why this is impossible.



Challenge: Mathematicians are constantly thinking about what is true and how can they convince people that it is true. The challenge is that the proof should be general, and not just an example that it works. Try to come up with an argument to convince yourself and other people that convex polygons are the only polygons we can obtain using the construction in 2. Hint: use drawings.