Optimization on polygons

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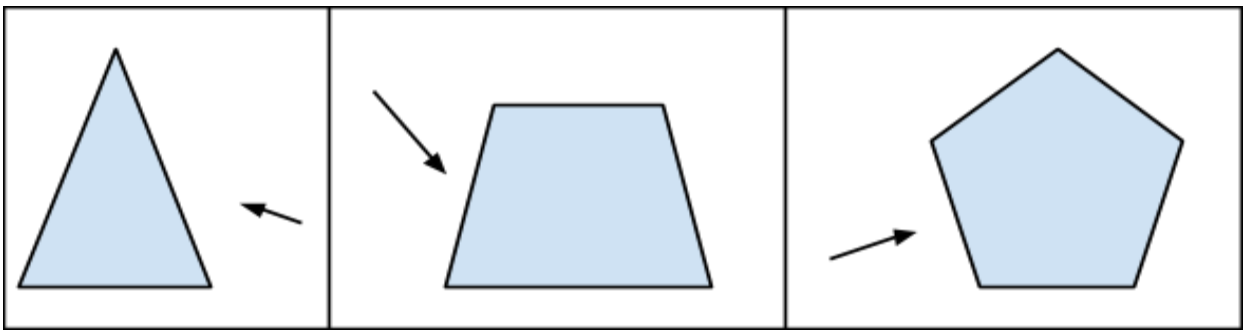
The goal of this worksheet is to become familiar with the connection between certain optimization problems and polygons. We will start with how to write polygons in terms of equations, move to inequalities and then to solving optimization problems. During the last session we saw how the problem of burning a hexagon can be connected to intersecting polygons and halfspaces. Let us practice doing this visually.

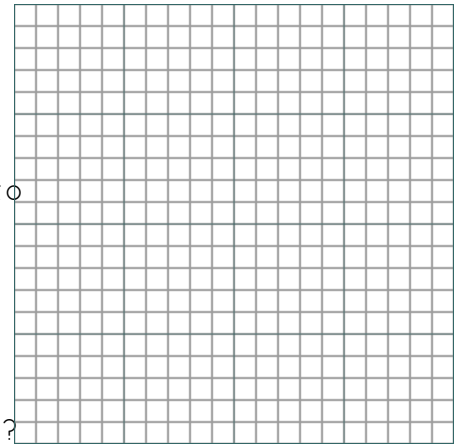
1. Look at the following drawings of polygons and directions in the plane. Highlight the face (vertex or edge) corresponding to this direction.



2. It is very difficult to follow this method and obtain an edge, most of the times if you pick a direction you will get a vertex. Try to explain why this is the case.

Last time some of you already started thinking about how to use equations to define a polygon. Let's now practice doing this. The good news is that we only need to be able to graph lines.

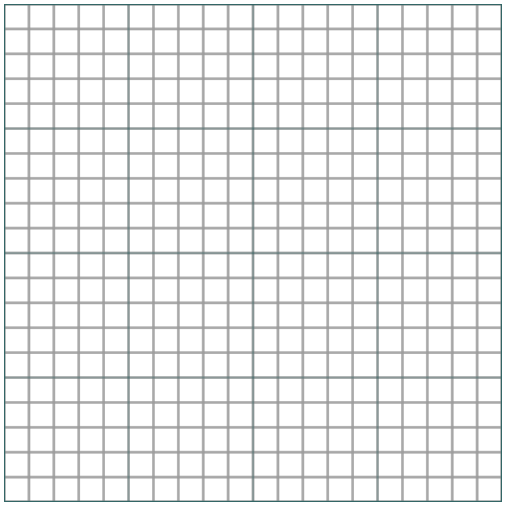
1. Do you remember how to graph lines? Try practicing a bit on the graph paper. For example, try graphing the lines:
    a. $y=6$
    b. $x=-2$
    c. $y=x+3$
    d. $y=1-2x$
3. Since we will be working with regions, we have to graph inequalities. Graph the following inequalities:
    a. $y\leq 6$
    b. $-2\leq x$
    c. $x+3\leq y$
    d. $1\leq 2x+y$
5. What object do you get after completing problem 3?
6. Now consider the direction going from the point (0,0) to the point (1,0), what face does this correspond to? Repeat the process with the direction from (0,0) to (1,2).



We are now finally ready to solve some optimization problems. We will now encounter some word problems that ask us to maximize the amount of money we can make. This

problems look intimidating unless we break them up into pieces. First, try to figure out the equations we will need to solve this. It is important to identify the direction we will want to optimize. Then graph them and find the face of the polytope in the maximizing direction.

6. A carpentry shop makes night and coffee tables. Each week the shop must complete at least 9 night tables and 13 coffee tables. The shop can produce at most 30 tables each week. If the shop sells night tables for $120 and coffee tables for $150, how many of each table should be produced for a maximum weekly income?
    a. What are the variables of this problem?
    b. What quantity is to be maximized and how do I express that quantity in terms of your unknowns? This will be the direction we will use to find a face of the polygon.
    c. The constraints of the problem are some inequalities that the variables satisfy. Write the inequalities for this problem and graph them.



    d. Translate your equation from b. to a direction and find the corresponding face. Congratulations! you just solved this optimization problem :).