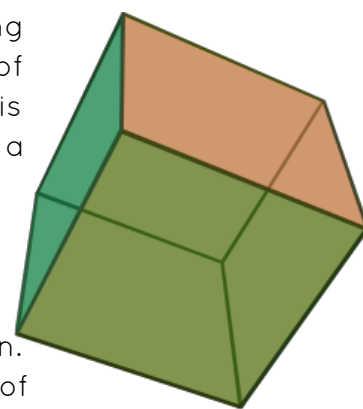


## Regular polyhedra

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Math Explorer's Club, Nov. 3, 2012

The goal of this worksheet is to understand regular polyhedra. According to wikipedia, a **regular polyhedron** is a polyhedron whose faces are congruent regular polygons which are assembled in the same way around each vertex. Can you think of examples? A cube is a great example. One could naively expect that if you pick any regular polygon, we can build a polyhedron having this polygon as faces. For example, all the faces of the cube are squares. We will see that this is not the case, actually we will classify all the regular polyhedra. The way we will use to verify this is a backward construction of the one we did at the beginning of the module.

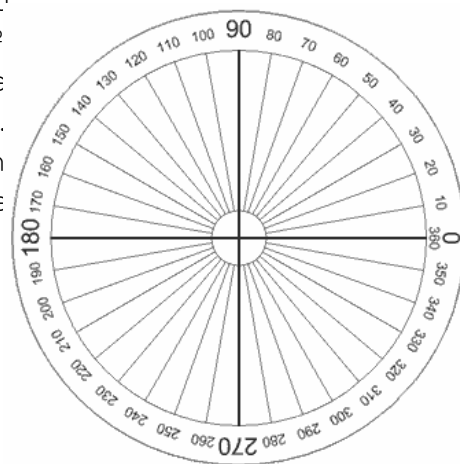
1. Let us start with a construction on (or better deconstruction of) the cube. Pick your favorite vertex on the cube and ignore everything except all the faces that contain the cube. Now think of cutting along your least favorite edge containing this vertex and open this corner to make it flat. Try drawing a cartoon of what you get.



2. This construction can be made with any polyhedron. Suppose now that we have a polyhedron made up of regular triangles with 3 triangles around each vertex. Suppose that we follow a similar process to the one on the cube, try drawing a cartoon of what you get.
3. How would you describe the cartoons you are getting?

To me, they look like a number of regular polygons glued together, lying on the plane, and sharing a common vertex. Moreover, they do not fill all the space around the vertex because this space is the space we need to fold our paper model and glue it.

4. Remember angle measurements? Recall that if you go once around a circle, you travel a total of  $360^\circ$ . The drawing on the left has the angle measurements as you go around the circle. Consider the drawings you made in 1, 2; what can you say about the sum of the angles around the vertex?

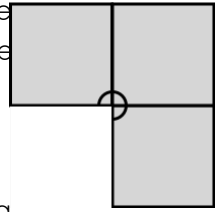


One of the many observations we can make is that the sum of the angles is less than

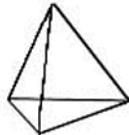
360°. If it were more than 360° we would not have space to fold.

5. Let's write a formula for the actual sum of the angles.

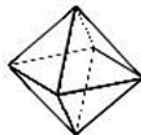
- a. Since the cube has 3 squares around each vertex, and the angle measurement of a square is 90°, write the sum of the angles around the vertex.



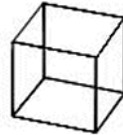
- b. What is the angle inside a regular triangle? Repeat exercise a. with 3 squares around each vertex.
- c. Now suppose that the number of regular  $n$ -gons around each vertex is  $k$  and the angle inside this  $n$ -gon measures  $A^\circ$ . What is the sum of the angles around such vertex?
- d. This sum must be  $< 360^\circ$ . Using trial and error figure out how large  $k$ , the number of regular  $n$ -gons, can be. (Hint: think about what happens with the angles inside an  $n$ -gon as the number of sides increases.)
- e. How small can  $k$  be? (Hint: remember we want to fold and get a 3D object.) Conclude what values  $k$  can have.
- f. What is the angle inside a regular  $n$ -gon? Use this and the inequality from d. to conclude how many sides your polygon can have.
- g. The last part would be to actually construct a polyhedron for each possible  $k$  and  $n$ . The drawing below illustrates the outcomes. Pair each regular polyhedron with the drawing.



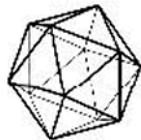
**Tetrahedron**



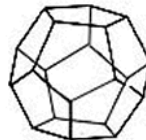
**Octahedron**



**Cube**



**Icosahedron**



**Dodecahedron**