Projective root systems, enhanced Dynkin diagrams and Weyl orbits

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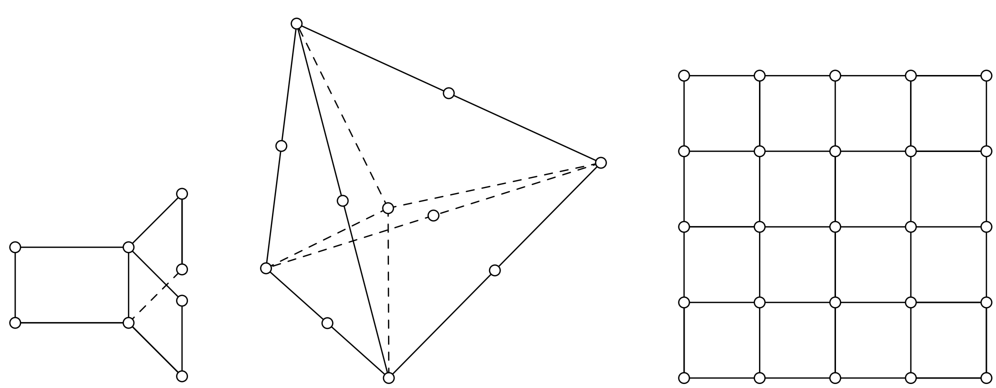
The talk is based on a joint paper with E. B. Dynkin which is now in progress. Our goal is to develop new tools for investigating classes $\mathcal{A}$ of conjugate semisimple subalgebras of semisimple Lie algebras, in particular, for a description of a natural partial order between such classes. (The relation $\mathcal{A}_1 \prec \mathcal{A}_2$ means that $A_1 \subset A_2$ for some $A_1 \in \mathcal{A}_1, A_2 \in \mathcal{A}_2$.)

One of tools is enhanced Dynkin diagrams which contain Dynkin diagrams as subdiagrams. As an example, the enhanced diagrams for E_6, E_7 and E_8 are placed below. [Nodes of the diagram for E_8 constitute 4×4 lattice on a torus.] Every node represents a pair of roots $(\alpha, -\alpha)$. We call these pairs projective roots. The Weyl group W acts on the set S of projective roots and therefore it acts on the space $\mathcal{V}$ of all subdiagrams of S isomorphic to Dynkin diagrams. Classes of conjugate regular subalgebras are in a 1-1 correspondence with Weyl orbits in $\mathcal{V}$.

A special role is played by maximal orthogonal subsets M of S. All of them are conjugate (like Cartan subalgebras). We denote W^M and we call the core group the group of all elements of W preserving M. The elements of a Weyl orbit in $\mathcal{V}$ which are contained in M form an orbit of W^M. This allows to reduce problems related to classes of conjugate regular subalgebras to similar problems regarding to orbits of the core group. No reduction of this kind is possible without a transition from roots to projective roots.



E_6 E_7 E_8