

Galois Theory of Parameterized Linear Differential Equations and Linear Differential Algebraic Groups

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Abstract: I will describe a Galois theory of differential equations of the form

$$\frac{\partial Y}{\partial x} = A(x, t_1, \dots, t_n)Y$$

where $A(x, t_1, \dots, t_n)$ is an $m \times m$ matrix with entries that are functions of the principal variable x and parameters $t_1, \dots, t_n$. The Galois groups in this theory are linear differential algebraic groups, that is, groups of $m \times m$ matrices $(f_{i,j}(t_1, \dots, t_n))$ whose entries satisfy a fixed set of differential equations. For example, in this theory the equation

$$\frac{\partial y}{\partial x} = \frac{t}{x}y$$

has Galois group

$$G = \{ (f(t)) \mid \frac{d^2(\log f)}{dt^2} = 0 \}.$$

We will give an introduction to the theory of linear differential algebraic groups and the above Galois theory and discuss the place of isomonodromic families in this theory.

This is joint work with Phyllis Cassidy.