How to throw accurately

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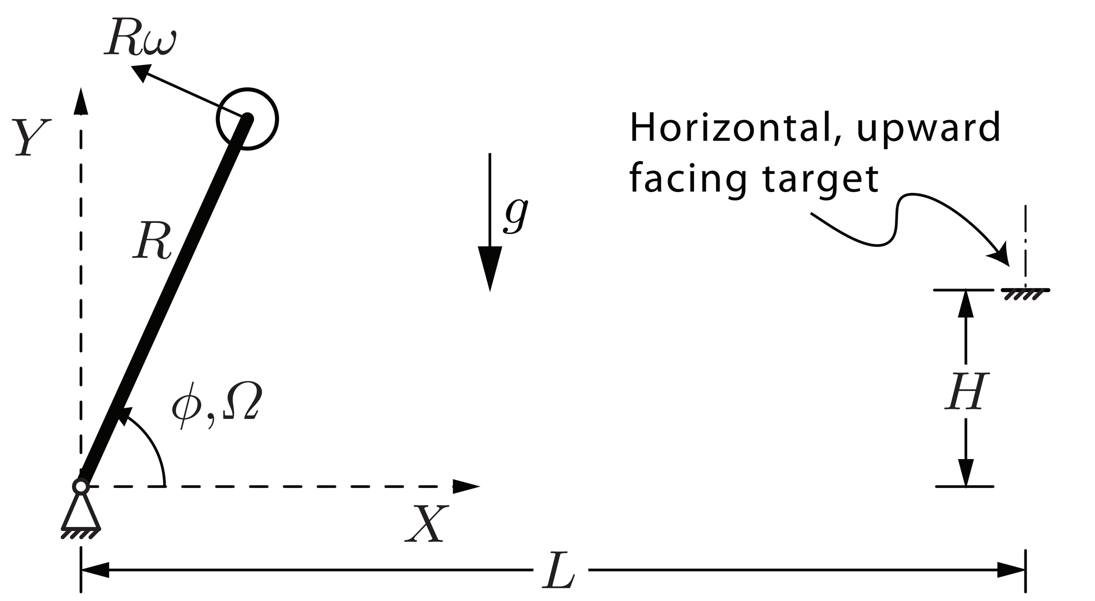
INTRODUCTION

How do the dynamics of projectile flight affect our choice between an overarm and underarm strategy?

- Throwing accuracy is important for:
  1. Motor development in children [1].
  2. Clinical evaluation of motor impairment [1].
  3. Hunting and human evolution [2].
  4. Sports performance [3].
- Launch angle and velocity of the projectile have to be carefully coordinated as errors in them are propogated by the dynamics of projectile flight.

MODEL OF THE ARM AS A HINGED BAR

'Super-human' arm with complete and independent control over the arm's release angle (ϕ) and angular velocity (Ω).



Dimensionless variables

$$x=\frac{X}{R},\ y=\frac{Y}{R}$$

$$t=T\sqrt{\frac{g}{R}},\ \omega=\Omega\sqrt{\frac{R}{g}}$$

$$l=\frac{L}{R},\ h=\frac{H}{R}$$

- Equations of motion:

$$\ddot{x}(t)=0,\ \dot{x}(0)=-\omega\sin\phi,\ x(0)=\cos\phi$$
$$\ddot{y}(t)=-1,\ \dot{y}(0)=\omega\cos\phi,\ y(0)=\sin\phi$$

LINEARIZED THROWING INACCURACY

There is a one-dimensional curve of release parameters $(\phi,\omega(\phi))$ for an accurate strike, and otherwise misses the target.

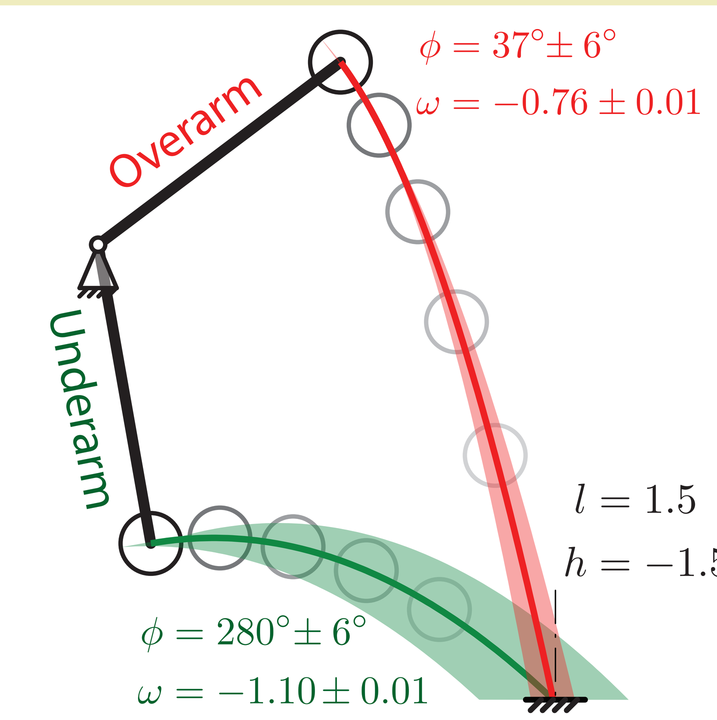
- Linearized map of input errors $(\delta\phi,\delta\omega)$ to targeting error (δx_h).

$$\delta x_h(\phi,\omega)\approx J_{\text{err}}(\phi_0)\begin{pmatrix}\delta\phi\\ \delta\omega\end{pmatrix},\quad J_{\text{err}}(\phi_0)=\left.\begin{pmatrix}\frac{\partial x_h}{\partial\phi} & \frac{\partial x_h}{\partial\omega}\end{pmatrix}\right|_{\substack{\phi=\phi_0\\ \omega=\omega_0(\phi_0)}}$$

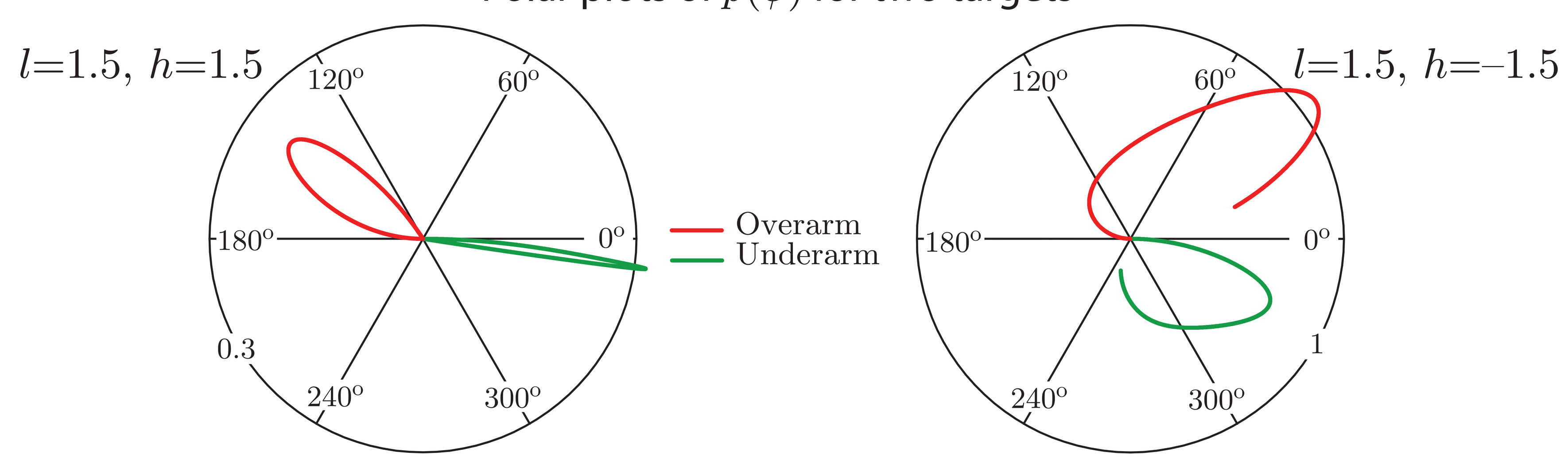
QUANTIFYING ERROR AMPLIFICATION

Error amplification is quantified by the singular value $\sigma(\phi)$ of the Jacobian $J_{\text{err}}(\phi)$ as a function of arm angle ϕ at release.

- Different throwing strategies amplify errors in release parameters differently.
- We grade strategies using the reciprocal of the singular value, $p(\phi)=1/\sigma(\phi)$.
- $p_{\text{o}}(\phi)$ and $p_{\text{u}}(\phi)$ are restrictions of $p(\phi)$ to overarm and underarm regimes.



Polar plots of $p(\phi)$ for two targets



- Underarm has a higher but narrower peak, i.e. it is more resilient to small noise, but less robust to large noise than overarm.
- The expected value of $p(\phi)$ for infinitesimal input noise is given by the peak of the distribution.
- Overarm is more resilient to both small and large input noise than underarm.
- The expected value of $p(\phi)$ for large input noise is given by the area of the distribution.

EXPECTED FRACTION OF OVERARM THROWS

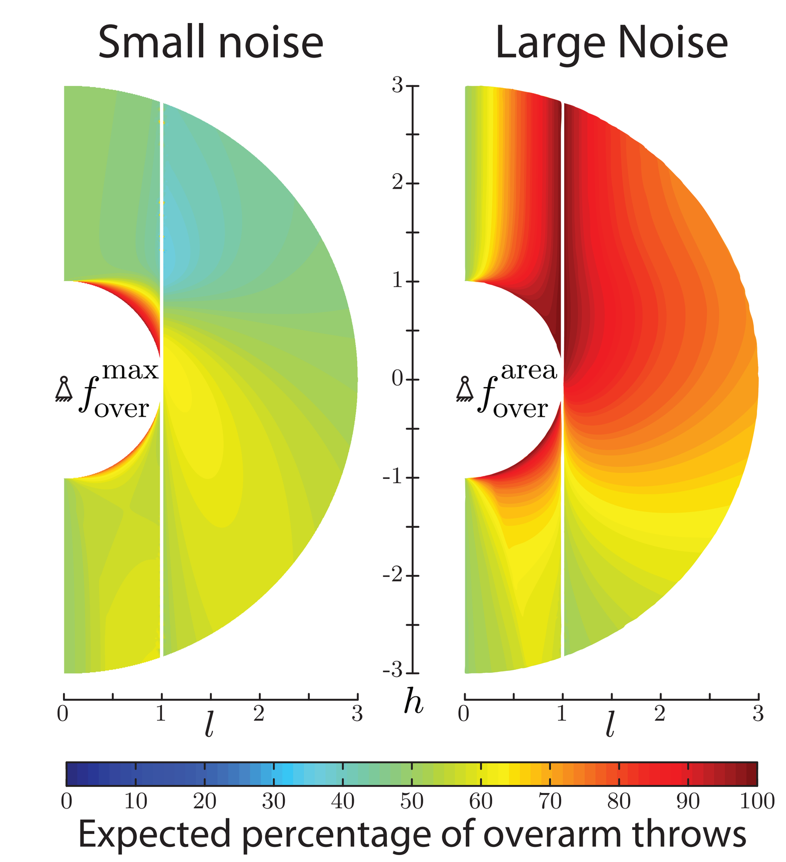
SMALL NOISE LIMIT

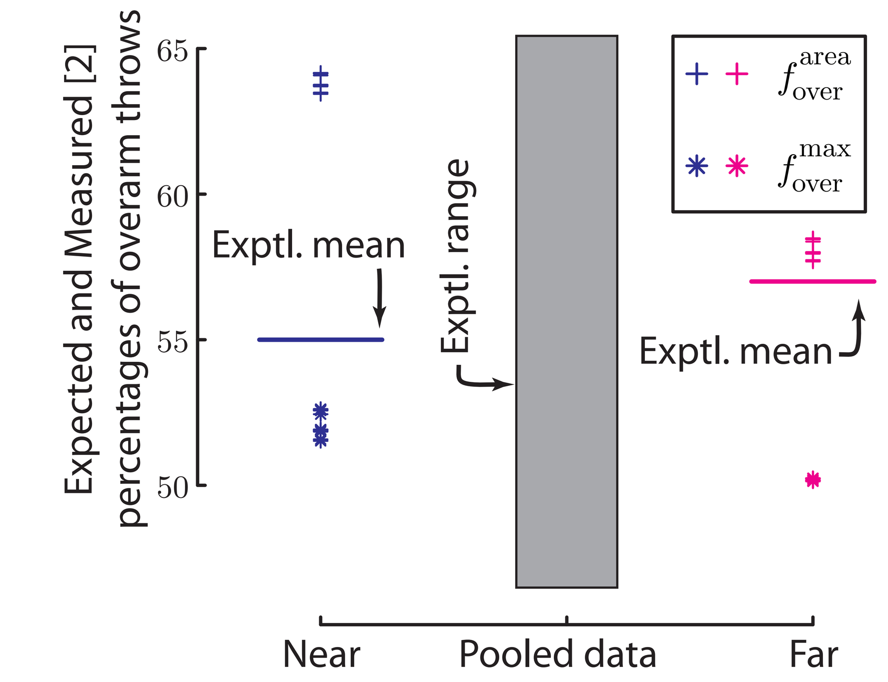
$$f_{\text{over}}^{\text{max}}=\frac{\max\limits_{\phi}p_{\text{o}}(\phi)}{\max\limits_{\phi}p_{\text{o}}(\phi)+\max\limits_{\phi}p_{\text{u}}(\phi)}$$

LARGE NOISE LIMIT

$$f_{\text{over}}^{\text{area}}=\frac{\int_{\Phi}p_{\text{o}}(\phi)d\phi}{\int_{\Phi}(p_{\text{o}}(\phi)+p_{\text{o}}(\phi))d\phi}$$

RESULTS & COMPARISON WITH DATA

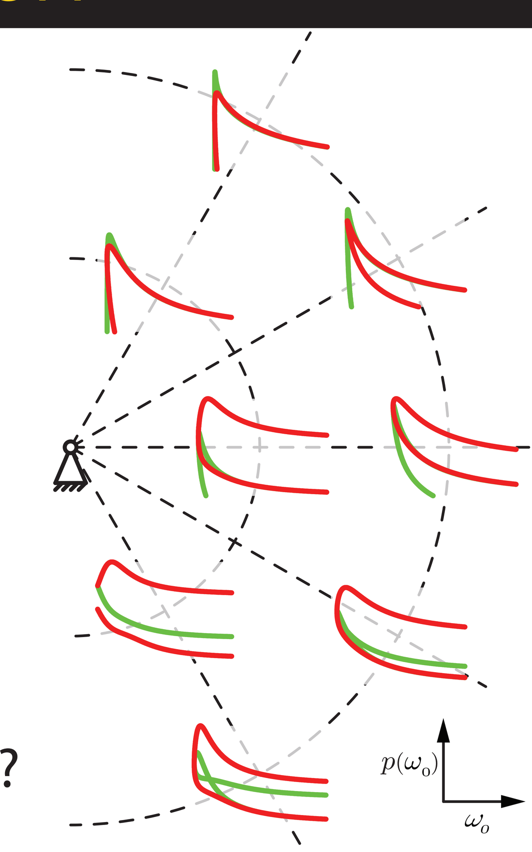




- Contour plots show distinct preferences for underarm and overam throws depending on target location.
- Recall that muscles have signal dependent noise [4].
- $f_{\text{over}}^{\text{max}}$ (∗) should and indeed predicts the nearer target.
- $f_{\text{over}}^{\text{area}}$ (+) should and indeed predicts the farther target.

SPEED-ACCURACY TRADE-OFF

- Speed-accuracy trade-off emerges solely from projectile dynamics.
- This is an alternative to current theories that rely on signal dependent noise in muscles [4].
- At higher speeds, like in hunting, the overarm is far superior. A pivotal role in human evolution, by creating a pressure to develop better overarm skills?



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