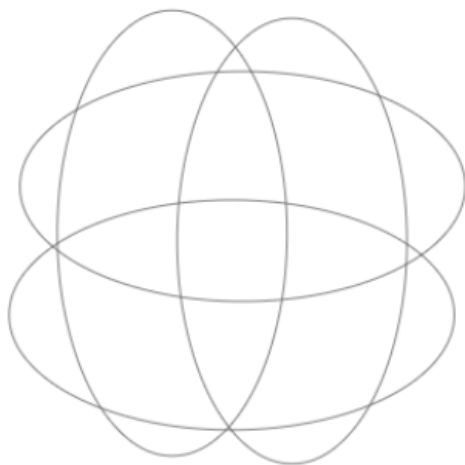# Exact Volumetric Helly Theorems

Undergraduate Math Club
CORNELL UNIVERSITY

SPEAKER

Alex Xue

ABSTRACT

Helly's theorem is a fundamental result about the intersection properties of convex sets. It states that given n convex sets in $\mathbb{R}^d$, if the intersection of every $d + 1$ convex sets is non-empty, then the intersection of all the convex sets is non-empty. In 1982, Bárány, Katchalski, and Pach proved a volumetric extension of Helly's theorem stating that if the intersection of every $2d$ of the convex sets has volume at least one, then the volume of the intersection of all the sets is at least d^{-2d^2}. This loss factor in the volume of the intersection is necessary. With additional constraints on the intersections, we can embed the convex sets in a higher dimension to obtain exact volumetric Helly results that avoid the loss factor. Other quantitative Helly-type theorems can be obtained with the same techniques. Joint work with Sherry Sankar and Pablo Soberón.

MAR 2 at 5:15pm

Malott 532 ⋆ Refreshments