Math 3340
Spring 2021
Lecture #15

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① Review for the 1^{st} prelim.

on the exam: up thru + including lecture #13
(Monday, Mar 8's lecture)

HW 1–4

book: chapters 2, 3

<u>exam</u>: open book: can use
- our text
- notes
- HW's + solns

should create a
"condensed info sheet"

What have we covered.

Ch 2: • functions
- equivalence relations
  + equiv. classes
  form a partition of the set
- $a \equiv b \pmod{n}$
  + division algorithm

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- permutations

  Can use: every element of S_n has a unique rep. (up to order) as a product of disjoint cycles.

  cycle notation

  sign of a permutation (even/odd)

- Chapter 3

concepts

- group G

  Abelian

  $|G|$

  $o(a)$

  a^n exponents

  [(warning: $(ab)^n = a^n b^n$ only if $ab = ba$!)]

- subgroups

  cosets aH, Ha, G/H

  $\rightarrow$ Lagrange + applications

  (mult tables)

examples

$\mathbb{Z}$

S_n, Sym S

$GL_n(\mathbb{F})$

$\mathbb{F} = \mathbb{R}, \mathbb{Q}$ or $\mathbb{C}$.

$SL_n(\mathbb{F})$

$\mathbb{Z}_n$, $\mathbb{Z}_n^\times$

$\langle a \rangle$ cyclic subgroup

$A_n \leq S_n$

$D_n \leq S_n$

$G \times H$

Aut G, Sym S

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- generating set ($S \subseteq G$)
  $\langle S \rangle \subseteq G$
- conjugacy: $a, b \in G$
  conjugate if $\exists\, g \in G$ $\quad gag^{-1} = b$.
  conjugacy classes
- $f: G \to H$ homomorphism
  isomorphism

ker f
im f $\Big\}$ subgroups

normal subgroup

simple group: G: only normal subgps are $\{e\}, G$.

not on exam:

$\mathbb{F}$ = field $\neq \mathbb{R}, \mathbb{Q}$ or $\mathbb{C}$.

3.7 : from 3.7.6 on. (ie: not G/ϕ)

3.8 : from pg. 177, anything involving G/N.

3.5 : did not cover in class

Last time:

- if $H \triangleleft G$, then $Ha = aH \quad \forall a \in G$
  (left cosets = right cosets)
- used this to define a group operation on G/H

$$(aH)(bH) = aHbH = abHH = (ab)H$$

showed: well-defined, satisfies the group axioms

- get a group G/H when $H \triangleleft G$.
- $|G/H| = |G|/|H| = [G:H]$.

Example $G = \mathbb{Z}$ (with $+$).

$$H = \langle n \rangle = n\mathbb{Z}$$
$$= \{kn : k \in \mathbb{Z}\}.$$

$H \leq G. \Longleftarrow$ so $H \triangleleft G$!

Question/aside if G is Abelian, $H \leq G$, when is $H \triangleleft G$? ALWAYS NORMAL.

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(a) what are the cosets G/H ?

(b) what is the group operation?

(c) $\mathbb{Z}/n\mathbb{Z}$ = ?? $\mathbb{Z}_n$!!

$\mathbb{Z}/n\mathbb{Z}$ $\quad H = n\mathbb{Z}$

cosets? not aH !

$$a + H = \{ a + h : h \in H \}$$

elem of $\mathbb{Z}/n\mathbb{Z}$

$$\begin{aligned}
0 + H &= \{ kn : k \in \mathbb{Z} \} = [0]_n \\
1 + H &= \{ 1 + kn : k \in \mathbb{Z} \} = [1]_n \\
2 + H & \quad \vdots \qquad\qquad \vdots \\
&\vdots \\
(n-1) + H &= \{ (n-1) + kn : k \in \mathbb{Z} \} = [n-1]_n
\end{aligned}$$

(b)

$$\begin{aligned}
(a + H) + (b + H) &= a + H + b + H \\
&= a + b + H + H \\
&= (a+b) + H
\end{aligned}$$

same as:

$$[a]_n + [b]_n = [a+b]_n$$

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⑥

So: $\mathbb{Z}/n\mathbb{Z} \simeq \mathbb{Z}_n$

(actually, equal!)