Homework # 1 Math 4310 Fall 2020

Due 9/9/20

Please submit your completed homework via gradescope on canvas. I encourage you to work with your classmates on this homework (except for the journal entries!) When you submit your work, please **list your collaborators**. (Your grade will not be affected.) Even if you work in a group, you should write up your solutions **yourself**! You should include all computational details, and proofs should be carefully written with full details. As always, please write **neatly and legibly** (feel free to use LaTeX to write up your solutions, if you wish!).

Reading. *The Secret to Raising Smart Kids* by Carol Dweck, in Scientific American. You can find it by Google, or in the files section on our canvas site (it is also linked to from the syllabus).

Journal entry. Every now and then, I will ask you to write the equivalent of a journal entry. For this first journal entry, please include a picture of yourself (if possible), include the name you would prefer me to address you by, and also the time zone you are in. I'd really like to hear about why you are considering this course and what you hope to get out of it, any reactions to the Dweck article, and also a short "mathematical biography" of yourself. Finally, if there were one thing that you would like me to know about yourself, what would that be?

Exercises.

1. Let $\mathbb{F}$ be a field. The **characteristic** of $\mathbb{F}$ is defined to be the smallest positive integer p such that $1 + 1 + \cdots + 1 = 0$, where there are p 1's in this formula. If no such sum is 0, then we say the characteristic of $\mathbb{F}$ is 0.

   (a) Find the characteristics of the fields $\mathbb{R}, \mathbb{C}, \mathbb{Z}_p$.

   (b) If $\mathbb{F}$ is a finite field, show that the characteristic p of $\mathbb{F}$ is not zero.

   (c) If $\mathbb{F}$ is a finite field, show that the characteristic p of $\mathbb{F}$ is a prime number.

2. In this problem we will investigate fields with 4 elements. Note: although your examples must satisfy all of the axioms for fields, you do not need to prove axioms (F2) (associativity) and (F5) distributivity (These are fairly timeconsuming and not so illuminating). But: make sure your examples satisfy these!

   (a) If $\mathbb{F}_4$ is a field with exactly 4 elements, what must the characteristic be? (justify your answer, of course! But you may use without proof statements from the previous problem).

   (b) Find a field $\mathbb{F}_4$ that has 4 elements. Write down the addition and multiplication tables of this field. Remember that two of your elements are 0 and 1!

   (c) Find all fields with 4 elements (i.e. write down all possible addition and multiplication tables. Your first two elements should be 0 and 1).

3. Let $\mathbb{Q}(\sqrt{5}) = \{a + b\sqrt{5} \mid a, b \in \mathbb{Q}\}$. Show that this set is a field.