Math 4310 Welcome !!

2 Sep 2020
Lecture 1

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(a) welcome, some questions for you

(b) mechanics of the course

canvas ⟶ syllabus
zoom, video links
HW
⋮

textbook(s) : Axler ⟵ main one
Friedberg, Insel + Spence ⟵ optional one

HW weekly neat + clear to read
latex'ed if you want.

grading

office hours TBD

TA : Joseph Fluegemann

feedback

(c) What is cool about this class?

Linear algebra has HUGE # of applications

machine learning
data analysis
computer graphics
data compression

(d) What do I expect you to know?

Proofs → clear understanding

Basic LA : matrices
$\mathbb{R}^n$
kernel + image (nullspace, range)
eigenvalues + vecs
determinants (2×2, 3×3).

What will we do? fields
vector spaces
linear maps
eigenvals + vecs + diagonalizability
inner products — least squares
SVD
Jordan canonical form

Fields

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before vector spaces – discuss scalars, e.g.: $\mathbb{R}$, $\mathbb{C}$, ? what else?

$\boxed{\text{example}}$ for breakout room:

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & -8 & 2 \end{bmatrix}$$

row reduced echelon form of A

(a) row reduce A

(b) what operations on scalars

want this entry to be 0

$$A \xrightarrow[R_2 \mathrel{+}= 2R_1]{} \begin{bmatrix} 1 & 3 & 5 \\ 0 & -2 & 12 \end{bmatrix} \xrightarrow[R_2 \mathrel{/}= -2.]{} \begin{bmatrix} 1 & 3 & 5 \\ 0 & 1 & -6 \end{bmatrix}$$

$$\xrightarrow[R_1 \mathrel{+}= -3R_2]{} \begin{bmatrix} 1 & 0 & 23 \\ 0 & 1 & -6 \end{bmatrix}$$

what operations on scalars are used?

multiplication
addition (subtraction)
division (or inverse of a scalar: $\frac{1}{2}$).
detect 0, 1 ... !

Def A **field** is a set $\mathbb{F}$ equipped with:

ⓐ 2 special elements $0, 1$, $0 \neq 1$.

ⓑ operation "addition" $+ : \mathbb{F} \times \mathbb{F} \longrightarrow \mathbb{F}$

$$(a, b) \longmapsto a+b$$

ⓒ multiplication: $\cdot \; \mathbb{F} \times \mathbb{F} \longrightarrow \mathbb{F}$

$$(a,b) \longmapsto a \cdot b \quad \text{or} \quad ab$$

$(\mathbb{F}, 0, 1, +, \cdot)$ must satisfy the following "axioms":

F1 commutativity of $+, \cdot$:

$\boxed{a+b = b+a}$, $\boxed{a \cdot b = b \cdot a}$ $\quad \forall a, b \in \mathbb{F}$

"for all"

F2 associativity of $+, \cdot$:

$\boxed{a + (b+c) = (a+b)+c}$ $\quad \boxed{a \cdot (b \cdot c) = (a \cdot b) \cdot c}$

$\forall a, b, c \in \mathbb{F}$.

F3 existence of identity elements

$\boxed{0 + a = a}$, $\boxed{1 \cdot a = a}$ $\quad \forall a \in \mathbb{F}$

F4 existence of inverses $\forall a \in \mathbb{F}$ $\quad \forall b \neq 0$ in $\mathbb{F}$

$\exists\, c, d \in \mathbb{F}$ s.t. $\boxed{a+c = 0}$ and $\boxed{b \cdot d = 1}$

there exists

$(c = (-a))$. $\quad (d = \frac{1}{b})$

F5 distributivity

$\boxed{a \cdot (b+c) = ab + a \cdot c}$ $\quad \forall a, b, c \in \mathbb{F}$