MATH 191 FALL 2002 PRELIM 2

Please write your name, your professor's name, and your section number on all of the exam booklets you use. **Show all your work** and put all your work in the exam booklet. Circle your final answers and be sure that you have explained them. Calculators, notes, and books are **NOT** permitted. **YOU DO NOT HAVE TO SIMPLIFY YOUR ANSWERS**. Good luck!!

1. (a) Find the following indefinite integrals.

   (i) (5 pts) $\displaystyle\int \frac{x}{\sqrt{1-2x^2}}\,dx$

   (ii) (5 pts) $\displaystyle\int x^2\sqrt{2-x}\,dx$

   (b) (15 pts) Find the position $s(t)$ of a particle that moves with acceleration $a = d^2s/dt^2 = t + \sin t \cos t$, subject to the conditions that $s = 1$ and $ds/dt = 2$ when $t = 0$.

2. (a) (10 pts) Find the area of the region enclosed by the curves $y = x^2 + 1$ and $y = \dfrac{x^2}{4} + 4$.

   (b) (15 pts) Find the volume of the solid generated by revolving the region in part (a) about the x-axis.

3. (10 pts) Find the *second* derivative of the function $f(x) = x\displaystyle\int_1^x \frac{\sin t}{t}\,dt$ $(x > 0)$.

4. (a) (10 pts) We want to use the trapezoid rule to evaluate the integral $\displaystyle\int_0^1 \frac{6x^2}{25}\,dx$. What is the minimum number of subintervals that we should have in our partition of $[0, 1]$ to ensure that our approximation is within .01 of the actual value?
   (**Note**: The absolute value $|E_T|$ of the error E_T of the trapezoid rule is less than or equal to $\frac{(b-a)}{12}h^2M$, where h is the step size and M is any upper bound for the values of $|f''|$ on the interval $[a, b]$).

   (b) (10 pts) Use the trapezoid rule with $n = 2$ to approximate the integral in (a).

   (c) (5 pts) Find the exact value of the integral in (a) and the exact value of $|E_T|$ in (b).

5. (a) (11 pts) Find the length of the curve $x = \dfrac{y^4}{4} + \dfrac{1}{8y^2}$ between $y = a$ and $y = 1$ where $0 < a < 1$.

   (b) (4 pts) What happens to this length as $a \to 1$ and as $a \to 0$?

Extra Credit Problems. You will get credit for **ONLY ONE** of the following problems.

1. (10 pts) Find the limit $\displaystyle\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n}\left(\frac{k}{n}\right)^4$.

2. (10 pts) A bowl is obtained by rotating the curve $x = f(y)$ about the y-axis as in the figure below. The bottom is determined by the plane perpendicular to the y-axis through the origin. For every $y > 0$, when the bowl is filled with water up to the level y, the volume of the water is $y + y^2$. What is $f(y)$?