Abstracts for SPDE meeting

Yuri Bakhtin

Title: Smooth densities for SPDE's

I will talk about Malliavin calculus for infinite-dimensional dynamics with polynomial nonlinearity and additive noise. I will describe conditions that guarantee existence and smoothness of densities for distributions of finite-dimensional projections of the solution. Joint work with Jonathan Mattingly.

Krzysztof Burdzy

Title: A particle representation for the heat equation solution

I will describe some hydrodynamic limit-type results for a particle representation of the heat equation solution. I will start with simulation results and conjectures. The talk will be mostly accessible to general mathematical audience and graduate students. Based on joint papers with Holyst, Ingerman, March and Quastel.

Don Dawson

Title: A caricature of mutually catalytic branching near dimension 2

Mutually catalytic branching involves a system with two types in which the branching rate of one depends on the presence of the other type. In R^1 it can be characterized as a weak solution of the pair of SPDE

$$\begin{aligned} dX_t(x) =& \frac{1}{2}\Delta X_t(x) + \sqrt{\gamma X_t(x) Y_t(x)}dW_t^1(x), \\ dY_t(x) =& \frac{1}{2}\Delta Y_t(x) + \sqrt{\gamma X_t(x) Y_t(x)}dW_t^2(x). \end{aligned}$$

In R^2 the two types have segregated spatial densities and all the action takes place at the interface. There exist technical difficulties which arise is studying this system and its interface behavior and extending it to higher dimensions - for example, moment measures do not characterize the law of the fixed time random measures. This talk will describe a caricature of this system by formulating it in the hierarchical group setting and analyzing the hierarchical mean-field limit. In this setting it is also possible to formulate a type of invariance principle. This is joint work in progress with Andreas Greven and Iljana Zähle based on Cox, Dawson and Greven (2004) and joint work with Andreas Greven, Frank den Hollander, Rongfeng Sun and Jan M. Swart on renormalization analysis.

Gautum Iyer

Title: Rare events in spatially extended media.

The dynamical behavior of many systems arising in physics, chemistry, biology, etc. is dominated by rare but important transition events between long lived states. Important examples include nucleation events during phase transition, conformational changes of macromolecules, or chemical reactions. Understanding the mechanism and computing the rate of these transitions is a topic that has attracted a lot of attention for many years. In this talk, I will discuss some recent theoretical developments for the description of rare events, as well as several computational techniques which allow to determine their pathways and rate. I will illustrate this concepts on the specific example of some reaction-diffusion equations driven by white-noise arising e.g. in the context of population dynamics and in the description of the kinetics of phase transitions.

Nick Krylov

Streamlining the analytic theory of SPDEs in domains

We present a "streamlined" theory of solvability of parabolic PDEs and SPDEs in half spaces in Sobolev spaces with weights. The approach is based on interior estimates for equations in the whole space and is easier than and quite different from the standard one.

Sergey Lototsky

Title: Estimating coefficients in stochastic parabolic equations.

Many natural processes can be modelled using a second-order parabolic equation driven by a space-time white noise. The general form of the operators in the equation is dictated by the underlying deterministic model, and the noise, either additive or multiplicative, is introduced to compensate for the model errors and short-time fluctuations. Typically, a realization of the process can be observed and the observations used to further specify the model, namely, to estimate the coefficients of the operators in the equation. In this talk, I will show how this estimation can be carried out even if both the observation time interval and the amplitude of noise are fixed.

Jonathan Mattingly

Title: Ergodicity of SPDE's

Govind Menon

Title: Smoluchowski's coagulation equations and Burgers turbulence

Smoluchowski derived his coagulation equations in 1917 to describe the accretion of colloids. His model has since been used to describe a variety of clustering processes (eg. the formation of smoke, dust and haze; the kinetics of polymerization; gravitational accretion). In the past fifteen years, this and other mean-field models of clustering have received considerable mathematical attention. An important feature is the formation of 'universal' distributions as mass is transported from small to large scales by clustering.

Burgers turbulence is the study of shocks in Burgers equation with random initial data or forcing. In the absence of forcing, shocks interact only by clustering. An elegant result of Carraro and Duchon (1994) and Bertoin (1998) reduces this problem to Smoluchowski's equation with additive kernel for a large class of random initial data.

I will describe the two models, the solution procedure that links them, and the consequent characterization of universality. A general theme is the fruitful interplay between aspects of the problem that are natural from the analytic and probabilistic points of view. This is joint work with Bob Pego (Carnegie Mellon).

Carl Mueller

Title: The speed of a random traveling wave

We describe joint work with Leonid Mytnik and Jeremy Quastel. The KPP equation is a standard model for the study of traveling waves. A large class of initial conditions yield solutions which converge to a limiting shape, which moves with constant velocity. Adding noise to the equation may give a stationary ensemble of shapes, with an average speed which is different than the speed of the deterministic wave. Brunet and Derrida have conjectured some surprising results about the speed of the wave in the random case, when the noise is small. Conlon and Doering have given an inequality which partially verifies half of the conjecture. We completely prove both halves of the conjecture. In addition, we give some further error terms which partially confirm a more recent conjecture of Brunet and Derrida.

Leonid Mytnik

Title: Uniqueness for Volterra-type stochastic equation

Let σ be a Hölder continuous function with index $\gamma \leq 1$ and set $\alpha \in (0, 1/2)$. Consider the following Volterra-type stochastic equation driven by Brownian motion B

$$X_t = X_0 + \int_0^t (t-s)^{-\alpha} \sigma(X_s) dB_s \,.$$

This equation can also be interpreted as a degenerate SPDE. We are interested in the set of parameters α, γ for which the pathwise uniqueness holds for the above equation. This is a joint work with Tom Salisbury.

Jim Nolen

Title: Front propagation in random drift

Some reaction-advection-diffusion equations admit traveling wave solutions. Similarly, solutions to the initial value problem may develop fronts propagating with a well-defined speed, even when the medium is random. In this presentation, I will describe recent work on front propagation in a random drift when the nonlinearity is the Kolmogorov-Petrovsky-Piskunov (KPP) type nonlinearity. The analysis makes use of large deviations estimates for the related diffusion processes in a random environment.

David Nualart

Title: Stochastic heat equation and intersection local times

In this talk we discuss the d-dimensional stochastic heat equation driven by a multiplicative Gaussian noise, which is white in space and it has the covariance of a fractional Brownian motion with Hurst parameter H in time. First we consider the equation in the Ito-Skorohod sense, and we show that the mild solution has an explicit development in the Wiener chaos. We provide sufficient conditions on the dimension d and the Hurst parameter H for the solution to exist. We prove that in the case $H > 1/2$ the square norms of the projections on each Wiener chaos are related to the moments of some weighted intersection local times of the d-dimensional Brownian motion. On the other hand, using an aproximation argument and Feynman-Kac's formula we obtain an expression for the moments of the solution in terms of the exponential moments of the weighted intersection local times. The case of an equation in the Stratonovich sense will be also discussed.

Philip Protter

Title: Complete and Incomplete Financial Markets, Martingales, and Bubbles

Abstract: We will present an abstract framework for mathematical finance theory, focusing on the two fundamental theorems of finance, and their implications. We will then show how a typically unasked question, when asked, results in a study of financial bubbles through the analysis of the nuanced differences of martingales and local martingales. Under the intuitive, but long overlooked, hypothesis of "no dominance" due to Merton, there are surprisingly no bubbles possible in complete markets, but they live, die, and are born or reborn naturally in incomplete markets, through a process roughly analogous to phase change in Ising models. The talk is based on joint work with Robert Jarrow and Kazuhiro Shimbo.

Boris Rozovski

Title: Wiener Chaos approach to SPDEs

Eric Vanden-Eijden

Title: Rare events in spatially extended media.

The dynamical behavior of many systems arising in physics, chemistry, biology, etc. is dominated by rare but important transition events between long lived states. Important examples include nucleation events during phase transition, conformational changes of macromolecules, or chemical reactions. Understanding the mechanism and computing the rate of these transitions is a topic that has attracted a lot of attention for many years. In this talk, I will discuss some recent theoretical developments for the description of rare events, as well as several computational techniques which allow to determine their pathways and rate. I will illustrate this concepts on the specific example of some reaction-diffusion equations driven by white-noise arising e.g. in the context of population dynamics and in the description of the kinetics of phase transitions.

Lorenzo Zambotti

Title: SPDEs with convex potentials

We want to present a new theory on gradient systems with convex potential in finite and infinite dimension. A general existence and stability result is given. The techniques are based on recent developments in the theory of gradient flows in the Wasserstein metric. Applications include reflecting processes in convex domains and convergence of fluctuations of stochastic interfaces.