

Some answers to Exploring Infinite Series Exercise – Math 112 – Fall 2001

1. $a_k = \frac{1}{k^p}$ where $p \leq 1$ is a sequence converging to 0, but the p -series with $p \leq 1$ diverges.
2. The answers for 3 will work.
3. Such a sequence doesn't exist because if the series converges, then the limit of the sequence must be 0.
4. Same as for 3.
5. By the ratio test, if the limit of $|\frac{a_{n+1}}{a_n}|$ as $n \rightarrow \infty$ is strictly less than one, then the series has to converge. So, if there is any hope for a positive sequence, it is when the limit is equal to one. The sequence $a_k = \frac{1}{k}$ has the desired properties.
Another possibility is to have a sequence which isn't positive. In this case the ratio of the terms could be negative and yet large enough in absolute value for the series to diverge. For example, $a_k = (-4)^k$ is a series where the ratio is -4 and the series diverges.
6. By the ratio test, if the limit of $|\frac{a_{n+1}}{a_n}|$ as $n \rightarrow \infty$ is strictly greater than one, then the series has to diverge. So, if there is any hope, it is when the limit is equal to one. ???
7. By the ratio test, if the limit of the ratio is greater than or equal to 9, the series would have to diverge. So the ratio cannot be greater than 9 for all terms of the sequence. So, there must be something different happening in the head and tail of the sequence. $a_k = 10^k$ for $k \leq 100$ will make the first condition true. However, this doesn't affect the convergence of the series. So pick the rest of the sequence to assure the series converges, like $a_k = \frac{1}{2^k}$ for $k > 100$.
8. $f(x) = \sin(2\pi x)$ does not have a limit as $x \rightarrow \infty$, but $a_k = \sin(2\pi k)$ converges to 0 (in fact it is equal to 0 for all k). Note: If the limit of the function is infinity, the the associated sequence will also have to approach infinity.