$\begin{array}{c} {\rm Project\ Based\ Math\ 112,\ Fall\ 2001}\\ {\rm Project\ \#2- While\ Visions\ of\ Roadrunners\ Danced\ in\ His\ Head}\\ {\rm Due\ Wednesday,\ November\ 7}\end{array}$

Wile E. Coyote Bleached Bones, AZ 59055 October 17, 2001

Math 112 Students Cornell University Ithaca, NY 14853

Dear Calculus Students,

HELP ME!! For the last two months, I've had a series of recurring nightmares that are about to drive me out of my mind! I talked to your enterprising and resourceful instructor, who referred me to you.

The scenario is nearly always the same. I'm standing at the end of a road that is 1000 meters long (the road has those little green kilometer markers on it), and there at the other end is that blasted Roadrunner, just standing there, sticking his tongue out! I start to go after him, but I can only run in slow motion, about 1 meter per second. After one second I'm 999 meters away from that pesky fowl, but then *the road stretches uniformly and instantaneously by 1000 meters* so I'm 1998 meters away!! I cannot speed up, so I'm still moving in slow motion, at 1 meter per second. After another second, the road stretches again by 1000 meters! And this just keeps on happening. Over, and over. And over. And over. Well, you get the idea. Then I wake up, hungry and frustrated.

I've gotta know: Do I ever get the silly bird, and if so, how long does it take? Should take a snack to eat along the way!

I can't take this much longer. I need an answer by November 7th, or I'll lose my brilliant, crafty mind!

Hungry as ever,

Wile E. Coyote

P.S. What happens if I have this dream and the road is a different length? Or I'm traveling at a different (yet constant) rate? Or the road stretches by some other (fixed) amount?

After reading Wile E. Coyote's sad tale, I have a couple of suggestions to help you get started. If you can solve the questions below, you'll have an answer for Wile E.

- 1. Suppose Wile E. is standing on road d meters long, with a meters ahead of him and b meters behind him. If the entire road stretches uniformly by c meters, how much of the road is now behind him?
- 2. After n seconds, Wile E. has taken n steps, and the road has just stretched instantly by 1000 meters yet again. How long is the road now?
- 3. Let d_n be the distance Wile E. is from his original position, after n steps and subsequent stretches.
 - (a) Find a formula for d_n in terms of d_{n-1} .
 - (b) Find a formula for d_n in terms of n, then prove it using mathematical induction.
- 4. Now use what you know to determine whether Wile E. travels far enough to reach the Roadrunner after some finite number of seconds n. If so, for what n does he get there? (Hint: you won't be able to guess and check to find an n value that works.)

Extra Credit: Answer Wile E.'s postscript plea.

As before, refer to the handouts "Instructions for Projects", and "A Guide to Writing in Mathematics Classes" for general guidelines on completing your project.