A Guide to Writing in Mathematics Classes

1 Why Should You Have To Write Papers In A Math Class?

For most of your life so far, the only kind of writing you've done in math classes has been on homeworks and tests, and for most of your life you've explained your work to people that know more mathematics than you do (that is, to your teachers). But soon, this will change.

Since you are in this calculus class, you know far more mathematics than the average American has ever learned—indeed, you know more mathematics than most college graduates remember. With each additional mathematics course you take, you further distance yourself from the average person on the street. You may feel like the mathematics you can do is simple and obvious (doesn't everybody know what a function is?), but you can be sure that other people find it bewildering. It becomes increasingly important, therefore, that you can explain what you're doing to others who might be interested: your parents, your boss, the media.

Nor are mathematics and writing far-removed from one another. Professional mathematicians spend most of their time writing: communicating with colleagues, applying for grants, publishing papers, writing memos and syllabi. Writing well is extremely important to mathematicians, since poor writers have a hard time getting published, getting attention from the Deans, and obtaining funding. It is ironic but true that most mathematicians spend more time writing than they spend doing math.

But most of all, one of the simplest reasons for writing in a math class is that writing helps you to learn mathematics better. By explaining a difficult concept to other people, you end up explaining it to yourself.

> *Every year, we buy ten cases of paper at $35 each; and every year we sell them for about $1 million each. Writing well is very important to us.*
> — Bill Browning, President of Applied Mathematics, Inc.

2 How is Mathematical Writing Different from What You've Done So Far?

A good mathematical essay has a fairly standard format. We tend to start solving a problem by first explaining what the problem is, often trying to convince others that it's an interesting or worthwhile problem to solve. On your homeworks, you've usually just said, "9(a)" and then plunged ahead; but in your formal writing, you'll have to take much greater pains.

After stating the problem, we usually state the answer immediately. Then we move on to showing how we arrived at this answer. Sometimes we even state the answer right along with the problem. It's uncommon to read a math paper in which the answer is left for the very end. Explaining the solution and then the answer is usually reserved for cases where the solution technique is even more interesting than the answer, or when the writers want to leave the readers in suspense. But if the solution is messy or boring, then it's typically best to hook the readers with the answer before they get bogged down in details.

It is important to explain as many of your mathematical derivations as possible in English words and complete sentences. For example:

To solve for x when $3x^2 - 21x + 30 = 0$, we used the quadratic formula, and found that either $x = 5$ or $x = 2$:

$$x = \frac{21 \pm \sqrt{21^2 - 4 \cdot 3 \cdot 30}}{2 \cdot 3} = \frac{21 \pm \sqrt{441 - 360}}{6} = \frac{21 \pm 9}{6} = \frac{30}{6} \text{ or } \frac{12}{6} = 5 \text{ or } 2.$$

Thus either $x = 5$ or $x = 2$.

Math is difficult enough that the writing around it should be simple. "Beautiful" math papers are the ones that are the easiest to read: clear explanations, uncluttered expositions on the page, well-organized presentation. For that reason, mathematical writing is not a creative endeavor the same way that, say, poetry is. You shouldn't be spending a lot of time looking for the perfect word, but rather should be developing the most clear exposition. Unlike students in humanities, mathematicians don't have to worry about over-using "trite" phrases in mathematics. In fact, Section 4 contains a list of trite but useful phrases that you may want to use in your mathematical papers. This guide, together with your checklist, should serve as a reference while you write. If you can master these basic areas, your writing may not be spectacular, but it should be clear and easy to read—which is the goal of mathematical writing, after all.

3 Following the Checklist

When you turn in your writing assignment, you should use a paper clip to attach the checklist to the front. You should use both the checklist and this booklet as a guide while you write because you will be graded directly on the criteria outlined on the checklist. What follows here is a more detailed explanation of the criteria that will be used for grading your papers.

1. Clearly restate the problem to be solved.

Do not assume that the reader knows what you're talking about. You don't have to restate every detail, but you should explain enough so that someone who's never seen the assignment can read your paper and understand what's going on without any further explanation from you. Outline the problem carefully.

2. State the answer in a complete sentence which stands on its own.

If you can avoid variables in your answer, do so. If not, remind the reader what they stand for. If your answer is at the end of the paper and you've made any significant assumptions, restate them. Do not assume that the reader has actually read every word and remembers it all (do you?).

3. Clearly state the assumptions which underlie the formulas.

For example, what physical assumptions do you have to make? (No friction? No air resistance? That something is lying on its side, or far away from everything else?) Sometimes things are so straightforward that there are no assumptions, but not often.

4. Provide a paragraph which explains how the problem will be approached.

It's not polite to plunge into mathematics without first warning your reader. Carefully outline the steps you're going to take, giving some explanation of why you're taking that approach. It's nice to refer back to this paragraph once you're deep in the thick of your calculations.

5. Clearly label diagrams, tables, graphs, or other visual representations of the math (if these are indeed used).

Even more than in literature, in math a picture is worth a thousand words, especially if it's well labeled! Label all axes with words if you use a graph. Give diagrams a title describing what they represent. It should be clear from the picture what any variables in the diagram represent. Make everything as clear and self-explanatory as possible. Warning: if you decide to draw pictures using a computer program, make sure to save them in such a way that they can be easily edited. You will probably need to go back and change some aspect before completing the project.

6. Define all variables used.

a) Even though you have labelled your diagram, you should still explain in words what your variables represent.

b) If there's a quantity you use only a few times, see if you can get away with not assigning it a variable. As examples:

Example (1):

Good: We see that the area of the triangle will be one-half of the product of its height and base that is, the area of the triangle is $(1/2) \times 3 \times 4 = 6$ square inches.
Not as good: We see that $A = \frac{1}{2}hb$, where A stands for the area of the triangle, b stands for the base of the triangle, and h stands for the height of the triangle, and so $A = (1/2) \times 3 \times 4 = 6$ square inches.

Example (2):

Good: Elementary physics tells us that the velocity of a falling body is proportional to the amount of time it has already spent falling. Therefore, the longer it falls, the faster it goes.
Not as good: Elementary physics tells us that $vt = g(t - t_0)$, where vt is the velocity of the falling object at time t, g is gravity, and t is the time at which the object is released. Therefore as t increases, so does vt: i.e., as time increases, so does velocity.

I hope that you'll agree that the first example of each pair is much easier to read.

c) The more specific you are, the better. State the units of measurement. Try to use words like "of", "from", "above". For example:

Good: We get the equation $d = rt$, where d is the distance from Sam's car to her home (in miles), r is the speed at which she's traveling (measured in miles per hour), and t is the number of hours she's been on the road.
Not as good: We get the equation $d = rt$, where d is the distance, r is the rate, and t is the time.

Avoid words like "position" (height above ground? sitting down? political situation?) and "time" (5 o'clock? January? 3 minutes since the experiment started?). Never mind that your instructor uses these words freely; you can too when you pass this class.

d) Variables in text should be italicized to tell them apart from regular letters.

7. Explain how each formula is derived, or where it can be found.

Don't pull formulas out of a hat, and don't use variables which you don't define. Either derive the formula yourself in the paper, or explain exactly where you found it (so other people can find it, too). Put important or long formulas on a line of their own, and then center them; it makes them much easier to read:

Good:

The total number of infected cells in a honeycomb with n layers is

$$1 + 2 + ... + n = n(n + 1)/2.$$

Therefore, there are $100(101)/2 = 5,050$ infected cells in a honeycomb with 100 layers.

Not as good:

The total number of infected cells in a honeycomb with n layers is $1 + 2 + ... + n = n(n + 1)/2$. Therefore, there are $100(101)/2 = 5,050$ infected cells in a honeycomb with 100 layers.

Most advanced word processors have an equation editor. If you don't have an equation editor, you may try formulas with tabs and keyboard characters, or you may wish to write the mathematics in by hand. All of these are fine options.

8. Give acknowledgment where it is due.

It's *extremely* important to acknowledge where your inspiration, your proofreading, and your support came from. In particular, you should cite: any book you look at, any computational or graphical software which helps you understand or solve the problem, any person you talk to (your instructor, or another projects instructor, in this case.) The more specific you are, the better.

9. Use correct spelling, grammar, and punctuation.

a) Don't forget, spelling and grammar are just as important in mathematics papers. Please spell-check and proofread your work for grammar mistakes. Better yet, ask a friend to read your paper.

b) Don't get sloppy with punctuation in sentences which contain mathematical formulas. Put a period at the end of the computation if it ends a sentence; use a comma if appropriate. An example follows.

> If Tweek's caffeine level varies proportionally with time, we see that
>
> $$C_t = kt,$$
>
> where C is his caffeine level t minutes after 7:35 a.m., and k is a constant of proportionality. We can solve to show that $k = 202$, and therefore his caffeine level by 11:02 ($t = 207$) is
>
> $$C_{207} = (202)(207) = 41,814.$$
>
> In other words, he's mightily buzzed.

c) Do not substitute mathematical symbols for English words (= and # are especially common examples of this). The symbol "=" is used only in mathematical formulas—not in sentences:

Good:
We let V stand for the volume of a single mug and n represent the number of mugs. Then the formula for the total amount of root beer we can pour, R, is $R = nV$.

Not as good:
We let V = volume of a single mug and n = the # of mugs. Then the formula for the total amount of root beer $R = nV$.

Bad:
We let V stand for the volume of the mug and n represent the number of mugs. Then the formula for the total amount of root beer we can pour, R, is R is nV.

d) Do, however, use equal signs when you state formulas or equations, because mathematical sentences need subjects and verbs, too.

Good:
Then the formula for the total amount of root beer we can pour is $R = nV$.

Not as good:
Then the formula for the total amount of root beer we can pour is nV.

10. Use correct mathematics.

This is self-explanatory.

11. Completely answered the original question.

So is this.

4 Good Phrases to Use in Math Papers:

- *Therefore* (also: *so, hence, accordingly, thus, it follows that, we see that, from this we get, then*)
- *I am assuming that* (also: *assuming, where, M stands for;* in more formal mathematics: *let, given, M represents*)
- *show* (also: *demonstrate, prove, explain why, find*)
- *This formula can be found on page 243 of Stewart's Calculus: Concepts and Contexts, single variable, Second edition, 2001.*
- *If you have any further questions, feel free to contact me or Sam Smart, who helped me develop this formula for you.*
- *While I am very glad to help you this time, you should be advised that my usual consultation fee is* \$85.

- *(see the formula above).* (also: *(see *), this tells us that . . .*)
- *if* (also: *whenever, provided that, when*)
- *notice that* (also: *note that, notice, recall*)
- *since* (also: *because*)

Remember to please proofread the final draft before you submit it. Please, please. No, really.

This handout was adapted from materials developed by Harel Barzilai, Annalisa Crannell, and "Guidelines for Projects" in Calculus: An Active Approach with Projects and Appendix B of Student Research Projects in Calculus.