MATH 6510, Algebraic Topology, Spring 2017
Homework 11, Due in class 1 May

Reading

- As you'll see in Question 3 below, the Borsuk–Ulam theorem can be deduced from the cup product structure on $H^*(\mathbb{R}P^n; \mathbb{Z}_2)$. Karol Borsuk was at the heart of an influential Polish school in topology. Stanislaw Ulam (also Wikipedia) had an extraordinarily wide ranging career. He made major contributions to the Manhattan Project, invented the Monte-Carlo method, and is one of the people credited with the discovery of cellular automata.
- Florian Frick tells me his favorite book is *Using the Borsuk–Ulam theorem* by Matousek.

Exercises

1. In class we determined the cup product $H^1(X) \times H^1(X) \to H^2(X)$ for surfaces X in the manner of Examples 3.7 and 3.8 on pages 207–8 of Hatcher (but we only used $\mathbb{Z}_2$ coefficients). Repeat these calculations (again, with $\mathbb{Z}_2$ coefficients) for $T = S^1 \times S^1$ (the case $g = 1$ in Hatcher's Example 3.7), and the Klein bottle (the case $g = 2$ in Hatcher's Example 3.8), but using Δ-complexes which are squares (with suitable boundary identifications), subdivided into two triangles by a diagronal.

2. Our second midterm included a question in which you were asked to show that $S^1 \times S^2$ and $S^1 \vee S^2 \vee S^3$ are not homotopy equivalent, and you were encouraged to answer this by considering their universal covers. Here's another approach: show that they have different cohomology rings.

3. Hatcher, page 229, Question 3.

4. Taking the product of the inclusion $\mathbb{R}P^1 \to \mathbb{R}P^\infty$ with itself n times gives a map from the n-torus T^n to $(\mathbb{R}P^\infty)^n$. Compute the induced map $H^*(T^n; \mathbb{Z}_2) \to H^*((\mathbb{R}P^\infty)^n; \mathbb{Z}_2)$. (See Hatcher, Theorem 3.12 on page 212 and Example 3.15 on page 217 for the cohomology ring structures.)