# MATH 6510, Algebraic Topology, Spring 2017
# Homework 4, Due in class 22 February

## Reading

- A proof of van Kampen's Theorem is on pages 44–46 of Hatcher.
- In categorical terms, the conclusion of van Kampen's Theorem is a push out in the category of groups.
- Where it all began.... here is John Stillwell's translation of Poincaré's *Analysis Situs* and here is a historical essay by Dirk Siersma.
- If you want to know all the details about the fundamental group of the Hawaiian Earring look at this paper by Cannon and Conner (journal version with pictures here).

## Exercises

1. Hatcher page 79, Qu. 6
2. (Weintraub Ex. 2.7.9) Prove directly the following special case of van Kampen's theorem: Let $X$ be the union $X = X_1 \cup X_2$ of two open sets $X_1$ and $X_2$. Suppose $X_1$, $X_2$ and $A = X_1 \cap X_2$ are all path-connected. Suppose that each of $X_1$ and $X_2$ is simply connected. Then $X$ is simply connected.
3. (Weintraub Ex. 2.7.9)
   (a) Let $C$ be a connected finite 1-complex. Show that the number of edges $n$ of $C$ not in a maximal tree $T$ is well-defined, i.e. independent of the choice of $T$.
   (b) Show that $C$ is homotopy equivalent to the $n$-leafed rose $R_n = \bigvee_n S^1$ for some $n$.
4. (Weintraub Ex. 2.7.9) Let $p$ be a prime. Show that the free group $\langle a, b\rangle$ has exactly $p + 1$ normal subgroups of index $p$.
5. (Hatcher) Construct an uncountable number of nonisomorphic connected covering spaces of $S^1 \vee S^1$. Deduce that a free group on two generators has an uncountable number of distinct subgroups. Is this also true of a free *abelian* group on two generators?
6. (Weintraub Ex. 2.7.12) Let $X$ be a path-connected space and let $f : (X, x_0) \to (X, x_0)$ be a map. The mapping torus $M_f$ of $f$ is the identification space $X \times I/\sim$ where $(x, 0)$ is identified to $(f(x), 1)$. Let $m_0$ be the image of the point $(x_0, 0)$ in $M_f$. Show that $\pi_1(M_f, m_0) = \pi_1(X, x_0) * \pi_1(S^1)/tgt^{-1} = f_*(g)$, where $t$ is the generator of $\pi_1(S^1)$.