413: PROBLEM SET 3. DUE THURSDAY 21 FEBRUARY

(1) Using the formula for the partial sums of a geometric series (or otherwise), check that the sequence of rational numbers whose n^{th} term is

$$x_n = \sum_{i=1}^{n} \frac{e_i}{10^i}$$

is a Cauchy sequence, for any $e_i \in \mathbb{N} \cup \{0\}$ with $0 \leq e_i \leq 9$. (The real number defined by this Cauchy sequence is usually denoted $0.e_1e_2e_3\ldots$)

(2) Section 2.3.3 #1.

(3) Section 2.3.3 #10.

(4) Section 2.4.5 #4.

(5) Use the triangle inequality to show that for any real numbers a and b, we have

$$||a| - |b|| \leq |a - b|.$$

Use this to show that if $\{x_n\}$ is a sequence of real numbers which converges to L, then the sequence $\{|x_n|\}$ converges to $|L|$.

(6) Find, if they exist, the supremum (least upper bound) and infimum (greatest lower bound) of the following subsets of $\mathbb{R}$.

- $\{1, 2, 3, 4\}$.
- $\{x \in \mathbb{Q} : x < \sqrt{2}\}$.
- $\mathbb{N}$.
- $\{1/n : n \in \mathbb{N}\}$.