413: PROBLEM SET 8. DUE THURSDAY 24 APRIL

(1) Section 6.1.5 #4.

(2) Section 6.1.5 #5.

(3) Section 6.1.5 #8.

(4) Section 6.2.4 #4.

(5) Section 7.2.4 #5. (In this problem, assume that a is rational.)

(6) Section 7.2.4 #9.

(7) Section 7.2.4 #10.

(8) Section 7.3.4 #12.

(9) Calculate the radius of convergence of the following power series:

    (a) $\sum_{n=0}^{\infty}(n^2 + 2n + 2)x^n$.

    (b) $\sum_{n=0}^{\infty}(3^n + (-3)^n)x^n$.

(10) Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} x + 2 & x \leq 0 \\ 4 - x & x > 0 \end{cases}$$

    Compute the Riemann integral $\int_{-1}^{1} f(x)dx$. Make sure you prove rigorously that your answer is correct.

(11) A *rational function* is a function of the form $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomial functions. For each of the power series $a(x)$ in Question 10, state whether $a(x)$ is equal to a rational function within its radius of convergence or not. If it is equal to a rational function, calculate the function.