Adjoint solution-based small-inclusion asymptotics of cost functions, with application to defect identification

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The identification of hidden flaws in solids and structures is a very challenging type of inverse problem. In this talk, we consider situations where overdetermined data is available on the boundary (e.g. measured displacement on a prescribed-traction surface). A standard approach to such problems is then to formulate and minimize a cost function expressing a misfit between the available observations and their simulation for assumed defect configurations. Such approach however leads to iterative minimization schemes that may entail very high computing costs due to repeated forward simulations, especially when considering 3-D configurations and dynamical measurements. Motivated by these considerations, we have been developing non-iterative approaches based on the concept of topological sensitivity, which consists in evaluating the perturbation undergone by the cost function due to the hypothetical nucleation of a small defect of prescribed nature and shape at a given sampling location. This perturbation can be evaluated as a function of the sampling location by a bilinear combination of two states (forward and adjoint), that can be computed by any of the standard numerical methods available. The coefficients of this combination are found from an analytical preparatory work where the cost function asymptotics is carried out, and depend on the assumed characteristics of the small defect through a polarization tensor. Qualitative defect identification is then achieved by evaluating the topological sensitivity as a function of sampling locations in a region of interest: lowest negative values of this indicator correspond to locations where the featured cost function will decrease most should a small defect be present there. Various numerical examples (based so far on simulated data) for identification of cavities or cracks 3-D acoustic or elastodynamic media show that this approach indeed provides useful qualitative results (location and number of defects), at a computing cost far lower than a full-fledged iterative inversion. Such results may be used either stand-alone or as good initial guesses for subsequent iterative procedures. A refinement of this asymptotic approach then consists in expanding the cost function in powers of the size of the trial small defect, beyond the previously-investigated leading contribution. This allows to set up approximate global search techniques, which give better defect size estimates than the previous "simple" topological sensitivity at a computing cost far lower than that entailed by usual global search algorithms. This approach, which requires a substantial analytical preparation work but is then easy to implement, has been so far formulated for impenetrable obstacles or penetrable inclusions in 3-D acoustic media. The formulation will be presented, and its potential and effectiveness demonstrated on numerical experiments.

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