(1) Characterize the Hamiltonians $H \in C(\mathbb{R}^d \times \mathbb{R}^d)$ the arise from finite horizon differential games. How does this change if we include a running cost. That is, if the payoff in response to controls $\mathbf{y}$ and $\mathbf{z}$ is

$$g(\mathbf{x}(T)) + \int_0^T h(\mathbf{x}(t), \mathbf{y}(t), \mathbf{z}(t))\, dt$$

for some $g \in C(\mathbb{R}^d)$ and $h \in C(\mathbb{R}^d \times \mathbb{R}^d \times \mathbb{R}^d)$.

(2) Show that u is upper semicontinuous if and only if, for all $K \subseteq U$ compacct and $g \in C(K)$, the difference $u - g$ attains its maximum on K.

(3) Show that, if $J^{0,+}_x u \cap J^{0,-}_x u \neq \emptyset$, then u is continuous at x.

(4) Show that, if $\varphi \in J^{1,\pm}_x u$ and $\varphi^+(x) = \varphi^-(x)$, then u is differentiable at x.

(5) Suppose u is the restriction of an element of $C^\infty(\mathbb{R})$ to the set $X = [0,1) \cup \{2\}$. Compute $J^{k,+}_j u$ for $k = 0, 1, 2$ and $j = 0, 1, 2, 3$.

(6) Prove Lemma 11.4 from the notes.

(7) Prove Corollary 12.3 from the notes.