Math6720s12 - Homework 2

Due: 7 March 2012 (Wednesday)

Homework is prefered typed, but legibly handwritten is accepted. The TA is allowed to mark problems with "take read, can't grade." Late homework is not accepted without prior consent.

Exercise 1. *(Bass 9.4)*

Exercise 2. *(Bass 9.6)*

Exercise 3. *(Bass 10.1)*

Exercise 4. *(Bass 10.4)*

Exercise 5. *(Bass 11.2)*

Exercise 6. *(Bass 11.3)*

Exercise 7. *(Øksendal 3.7) A famous result of Ito (1951) gives the following formula for the n times iterated Ito integral:*

$$n! \int \cdots \int_{0 \le u_1 \le \cdots \le u_n \le t} dW_{u_1} \cdots dW_{u_n} = t^{\frac{n}{2}} h_n \left(\frac{W_t}{\sqrt{t}} \right). \tag{1}$$

Where h_n is a Hermite polynomial of degree n.

(1) *Verify that in each of these n Ito integrals the integrand satisfies the necessary properties.*
(2) *Verify the equation for $n = 0, 1, 2$ by hand.*
(3) *What is the covariance of $h_n(W_t)$ and $h_m(W_t)$? Are they independent?*

You may, with proper attribution, quote any results about Hermite polynomials as you need.