0916 - DIS 202

Tuesday, September 15, 2020 7:39 PM

1. Find all partials and second partials of $f(x, y) = 3xy - x^2 - y^2$ at the origin.

$\frac{\partial f}{\partial x} = 3y - 2x$ $\quad f(x,y) = f(y,x)$

$\frac{\partial f(x,y)}{\partial y} = \frac{\partial f(y,x)}{\partial y} = [3y-2x]_{x,y \text{ exchanged}} = 3x - 2y$

$\frac{\partial^2 f}{\partial x^2} = -2 = \frac{\partial^2 f}{\partial y^2} \quad \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}(3x-2y) = 3 = \frac{\partial}{\partial y}(3y-2x) = \frac{\partial^2 f}{\partial y \partial x}$

2. What is the plane tangent to $f(x, y) = x^3 - 3xy^2$ at $(2, 1, 2)$?

$\left.\frac{\partial f}{\partial x}\right|_{(2,1,2)} = 3x^2 - 3y^2 \Big|_{(2,1,2)} = 9$

$\left.\frac{\partial f}{\partial y}\right|_{(2,1,2)} = -6xy \Big|_{(2,1,2)} = -12$

$(2,1,2) + t\cdot(1,0,9) + k(0,1,-12)$

$9x - 12y = z + C$

$18 - 12 = 2 + C$

$\Rightarrow C = 4$

3. What is the plane tangent to

$$f(x, y) = \frac{7xy}{e^{x^2+y^2}}$$

$f(x,y) = f(y,x)$

① at $x = 1, y = 1$?

② When is the tangent plane parallel to the xy-plane?

① $\frac{\partial f}{\partial x} = \frac{e^{x^2+y^2}\cdot 7y - 7xy\cdot 2x\cdot e^{x^2+y^2}}{e^{2(x^2+y^2)}}$ $\quad x=1, y=1 \quad \frac{7e^2 - 14e^2}{e^4} = \frac{-7}{e^2}$

$\Downarrow$ x,y exchanged

$\frac{\partial f}{\partial y} = \frac{e^{x^2+y^2}\cdot 7x - 7yx\cdot 2y\, e^{x^2+y^2}}{e^{2(x^2+y^2)}}$ $\quad x=1, y=1 \quad \frac{7e^2 - 14e^2}{e^4} = \frac{-7}{e^2}$

$x=1, y=1$

$z = \frac{7}{e^2}$

$\frac{-7}{e^2}x + \frac{-7}{e^2}y - z = C$

$\frac{7}{e^2}x + \frac{7}{e^2}y + z = \frac{21}{e^2}$

$$\boxed{\frac{7}{e^2}(x-1) + \frac{7}{e^2}(y-1) + (z - f(1,1)) = 0}$$

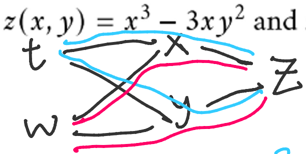
② $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0 \quad e^{2(x^2+y^2)} \neq 0$

$e^{x^2+y^2}(7y - 7\cdot 2\cdot x^2 y) = e^{x^2+y^2}\cdot 7y(1 - 2x^2)$

$\frac{\partial f}{\partial y} : e^{x^2+y^2}\cdot 7x(1-2y^2)$

$$\boxed{\begin{array}{c} x=0 \quad y=0 \\ 2x^2 = 2y^2 = 1 \end{array}}$$

4. If $z(x,y) = x^3 - 3xy^2$ and $x = t^2 - w^2$, $y = t + w$, what is $\frac{\partial z}{\partial t}$? What is $\frac{\partial z}{\partial w}$?



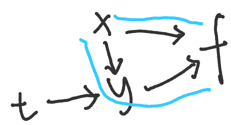
$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t} = (3x^2 - 3y^2)\cdot 2t + (-6xy)\cdot 1$$

$$\frac{\partial z}{\partial w} = \frac{\partial z}{\partial x}\,\frac{\partial x}{\partial w} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial w} = (3x^2 - 3y^2)\cdot(-2w) + (-6xy)\cdot 1$$

5. If $f(x,y) = xy^2 - x^2y$, $y(x,t) = t + \sin x$, what is $\frac{\partial f}{\partial x}$?

What is $\frac{\partial^2 f}{\partial x^2}$?

$$\text{"}\frac{\partial f}{\partial x}\text{"} : \lim_{h\to 0} \frac{f(x+h, y(x+h)) - f(x,y)}{x+h-x}$$

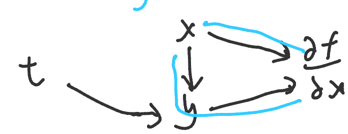


$$\frac{\partial f}{\partial x} = \left.\frac{\partial f}{\partial x}\right|_{y \text{ fixed}} + \left.\frac{\partial f}{\partial y}\right|_{x \text{ fixed}} \cdot \frac{\partial y}{\partial x}$$

$$= y^2 - 2xy + (2xy - x^2)\cdot\cos x$$

$$f(x, t+\sin x) = x(t+\sin x)^2 - x^2(t+\sin x)$$

$$\frac{\partial f}{\partial x} = \underbrace{(t+\sin x)^2}_{y^2} + \underbrace{2x(t+\sin x)\cdot\cos x}_{2xy\cos x} \underbrace{- 2x(t+\sin x)}_{-2xy} \underbrace{- x^2\cdot\cos x}_{-x^2\cos x}$$



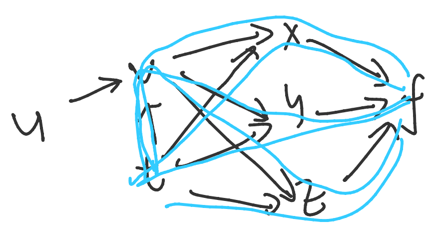
$$\frac{\partial^2 f}{\partial x^2} = -2y + (2y - 2x)\cdot\cos x - (2xy - x^2)\sin x + \left(2y - 2x + (2x)\cos x\right)\cos x$$

$$\frac{\partial}{\partial x}\cdot\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)_{y\text{-fixed}} + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)_{x\text{-fixed}}\cdot\frac{\partial y}{\partial x}$$

6. If we have $f(x,y,z)$, $x(v,t)$, $y(v,t)$, $z(v,t)$, $v(u,t)$, what does the dependencies look like?

What is $\frac{\partial f}{\partial t}$?

$$\left.\frac{\partial f}{\partial x}\right|_{y,z} \left.\frac{\partial x}{\partial v}\right|_{t} \left.\frac{\partial v}{\partial t}\right|_{u} + \left.\frac{\partial f}{\partial x}\right|_{y,z} \left.\frac{\partial x}{\partial t}\right|_{v}$$



+ replace x by y + replace x by z