0918 - DIS 202

Thursday, September 17, 2020 9:27 PM

1. For a function $f(x, y)$, if f_x and f_y both exists at $(1,2)$, then for any unit vector $\vec{u} = (a, b)$,

$$D_u f(1,2) = f_x(1,2)a + f_y(1,2)b.$$

May not be differentiable around $(1,2)$ $\rightarrow$ $\boxed{\text{false.}}$

if f_x, f_y both exist in a region around $(1,2)$ then becomes true

2. Consider $f(x_1, x_2, x_3)$.

$$D_v\left(\frac{\partial f}{\partial x_1}\right) = \frac{\partial^2 f}{\partial x_2 \partial x_1} \qquad \text{when} \qquad \vec{v} = \langle 0, 1, 0\rangle.$$

$\vec{v} = \langle 0, 1, 0 \rangle$ $\quad g(x_1, x_2, x_3)$ $\quad D_v(g) = \frac{\partial}{\partial x_2} g$

$\vec{u} = \langle 1, 0, 0 \rangle$ $\quad D_u(g) = \frac{\partial}{\partial x_1} g$ $\quad \vec{w} = \langle 0, 0, 1 \rangle$ $\quad D_u(g) = \frac{\partial}{\partial x_3} g$

$g = \frac{\partial f}{\partial x_1}$ $\quad \boxed{D_v(g) = \frac{\partial}{\partial x_2} \frac{\partial f}{\partial x_1}}$

True.

$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + \frac{1}{x_3}$

for any fixed $x_3 \neq 0$

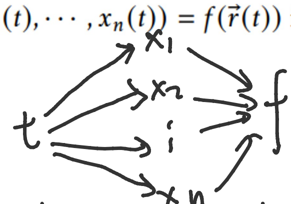
if asking $D_v\left(\frac{\partial f}{\partial x_1}\right) \overset{?}{=} \frac{\partial^2 f}{\partial x_1 \partial x_2}$

need $f(x_1, x_2, x_3)$ diff'able in (x_1, x_2) for fixed x_3

$f(x_1, x_2, c) = x_1^2 + x_2^2 + \frac{1}{c}$ is differentrable

$f(x_1, x_2, x_3, x_4, x_5, x_6)$ fix x_3, x_4, x_5, x_6 see if $f(x_1, x_2, c, d, e, f)$ diff'ble

3. If $f(x_1(t), x_2(t), \cdots, x_n(t)) = f(\vec{r}(t))$ is a function of n variables, and each variable x_i is a function of t. Then



$$\left.\frac{\partial f}{\partial t}\right|_{t=t_0} = \vec{\nabla} f(t_0) \cdot \vec{r}\,'(t_0).$$

$\vec{r}(t)$: a curve

$f(x, y, z)$ $\quad \vec{\nabla} f = \langle f_x, f_y, f_z \rangle$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial f}{\partial x_2}\frac{dx_2}{dt} + \cdots + \frac{\partial f}{\partial x_n}\frac{dx_n}{dt}$$

$\vec{r}(t) = \langle x_1, x_2, \cdots, x_n \rangle$

$\vec{\nabla} f = \left\langle \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \cdots, \frac{\partial f}{\partial x_n} \right\rangle$ $\quad \boxed{\text{True}}$

$r'(t) = \langle x_1'(t), x_2'(t), \cdots, x_n'(t) \rangle$

4. What is the gradient vector of

$$f(x, y, z) = x^2y^4 + \cos(z + y^2) - 7xz?$$

$\nabla f = \langle f_x, f_y, f_z \rangle$

$= \langle 2xy^4 - 7z, 4x^2y^3 + 2y(-\sin(z+y^2)), -\sin(z+y^2) - 7x \rangle$

5. Find the directional derivative of

$$g(r, \theta) = r^2 - r\cos(4\theta)$$

in the Cartesian direction of $(-1, 0)$ at $r = 3, \theta = \frac{\pi}{2}$.

$g: \mathbb{R}^2 \to \mathbb{R}$ $\quad x = r\cos\theta$

$g(r,\theta) = f(r\cos\theta, r\sin\theta)$ $\quad y = r\sin\theta$

$r\frac{\partial g}{\partial r} = \frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial r} = \frac{\partial f}{\partial x} r\cos\theta + \frac{\partial f}{\partial y} r\sin\theta$

$\frac{\partial g}{\partial\theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial\theta} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial\theta} = \frac{\partial f}{\partial x}(-r\sin\theta) + \frac{\partial f}{\partial y}\cdot r\cos\theta$

$yr\frac{\partial g}{\partial r} = \frac{\partial f}{\partial x}\cdot xy + \frac{\partial f}{\partial y}\cdot y^2$

$x\frac{\partial g}{\partial\theta} = \frac{\partial f}{\partial x}(-y)x + \frac{\partial f}{\partial y}\cdot x^2$

$r\frac{\partial f}{\partial y} = \left(yr\frac{\partial g}{\partial r} + x\frac{\partial g}{\partial\theta}\right)\cdot\frac{r}{x^2+y^2}$

$= \frac{yr}{r}\frac{\partial g}{\partial r} + \frac{x}{r}\frac{\partial g}{\partial\theta}$ $\quad r = \sqrt{x^2+y^2}$

$= r\sin\theta\frac{\partial g}{\partial r} + \cos\theta\frac{\partial g}{\partial\theta}$

$r\frac{\partial f}{\partial x} = r\cos\theta\, g_r - \sin\theta\, g_\theta$

$3\frac{\partial f}{\partial x} = -\sin\theta\, g_\theta = -g_\theta$ $\quad \frac{\partial f}{\partial x} = -\frac{1}{3}g_\theta$

2nd way of thinking about it: