0918 - DIS 204

Thursday, September 17, 2020 9:27 PM

If you prefer to go in a breakout room to collaborate on HW or work by yourself, just click "raise hand" or message me.
The breakout room will stay open until 10am.

1. For a function $f(x, y)$, if f_x and f_y both exists at $(1, 2)$, then for any unit vector $\vec{u} = (a, b)$,

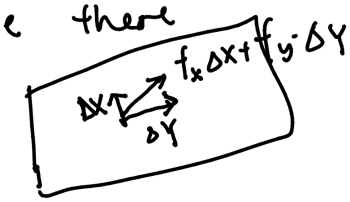
$$D_u f(1,2) = f_x(1,2)a + f_y(1,2)b.$$

$\boxed{\text{False}}$

May not be differentiable at $(1, 2)$

f_x, f_y both exist in a region around $(1, 2)$

$\Rightarrow$ f is differentiable there



2. Consider $f(x_1, x_2, x_3)$.

$$D_v\left(\frac{\partial f}{\partial x_1}\right) = \frac{\partial^2 f}{\partial x_2 \partial x_1} \quad \text{when} \quad \vec{v} = \langle 0, 1, 0 \rangle.$$

$\vec{v} = \langle 0, 1, 0 \rangle$ unit vector in x_2 direction

$\boxed{\text{True}}$

$D_v(g) = \frac{\partial}{\partial x_2} g$

$D_v\left(\frac{\partial f}{\partial x_1}\right) = \frac{\partial}{\partial x_2} \frac{\partial f}{\partial x_1} = \frac{\partial^2 f}{\partial x_2 \partial x_1}$

3. If $f(x_1(t), x_2(t), \cdots, x_n(t)) = f(\vec{r}(t))$ is a function of n variables, and each variable x_i is a function of t. Then

$$\left.\frac{\partial f}{\partial t}\right|_{t=t_0} = \vec{\nabla} f(t_0) \cdot \vec{r}'(t_0).$$

$\boxed{\text{True}}$

$\vec{\nabla} f = \langle f_{x_1}, f_{x_2}, \cdots, f_{x_n} \rangle$

$r'(t) = \langle x_1'(t), x_2'(t), \cdots, x_n'(t) \rangle$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial f}{\partial x_2}\frac{dx_2}{dt} + \cdots + \frac{\partial f}{\partial x_n}\frac{dx_n}{dt}$$

4. What is the gradient vector of

$$f(x, y, z) = x^2y^4 + \cos(z + y^2) - 7xz?$$

$$\vec{\nabla} f = \langle f_x, f_y, f_z \rangle$$

$$= \langle 2xy^4 - 7z, 4x^2y^3 - 2y\sin(z+y^2), -\sin(z+y^2) - 7x \rangle$$

5. Find the directional derivative of

$$g(r, \theta) = r^2 - r\cos(4\theta)$$

we can calculate g_r g_θ

in the Cartesian direction of $(-1, 0)$ at $r = 3, \theta = \frac{\pi}{2}$.

$$g : \mathbb{R}^2 \to \mathbb{R}$$

$$g(r,\theta) = f(r\cos\theta, r\sin\theta)$$

$$= f(x,y)$$

$x = r\cos\theta$

$y = r\sin\theta$

$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$

$$r\frac{\partial g}{\partial r} = \frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial r} = \frac{\partial f}{\partial x} r\cos\theta + \frac{\partial f}{\partial y} r\sin\theta$$

$$\frac{\partial g}{\partial \theta} = \frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial \theta} = \frac{\partial f}{\partial x}(-y) + \frac{\partial f}{\partial \theta} x$$

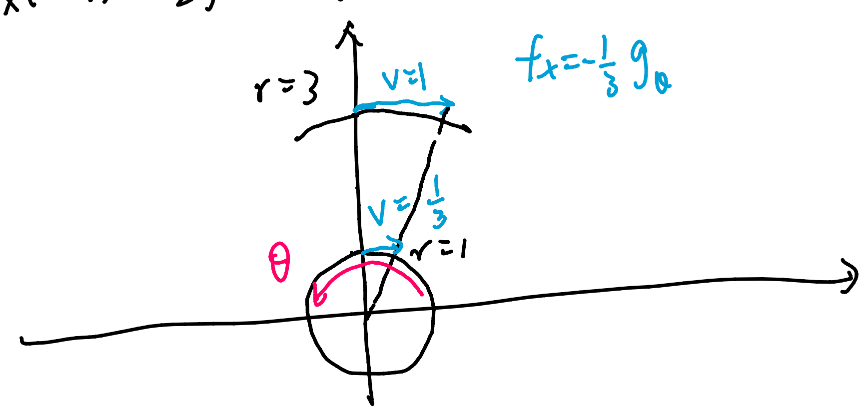
$$\begin{cases} yr\, g_r = f_x yx + f_y y^2 \\ x g_\theta = f_x(-yx) + f_y x^2 \end{cases}$$

want to write f_x, f_y in terms of g_r, g_θ

Add to get $f_y = \frac{1}{r^2}(yrg_r - xg_\theta)$ Note $r^2 = x^2 + y^2$

$f_x = \frac{1}{r^2}(xrg_r - yg_\theta)$ $\quad rf_x = xg_r - \frac{y}{r}g_\theta = r\cos\theta\, g_r - \sin\theta\, g_\theta$

$3f_x(r=3, \theta=\frac{\pi}{2}) = -g_\theta(3, \frac{\pi}{2})$ $\iff$ $f_x = -\frac{1}{3}g_\theta$ $\quad \theta = \frac{\pi}{2}$ $\quad r = 3$



$f_x = -\frac{1}{3}g_\theta$

6. (a) What is the directional derivative of

$$f(x, y) = \frac{1}{y - x^2} \text{ at } (-2, 3)$$

in the direction of $u = \langle 1, 2 \rangle$?

(b) In which direction is the rate of change maximized?

(a) $f_x = \dfrac{2x}{(y-x^2)^2}$ $\quad f_y = \dfrac{-1}{(y-x^2)^2}$

rational functions : check $(y-x^2)^2 \neq 0$ for some region around $(-2,3)$

$3-(-2)^2 = -1 \neq 0$

$\forall\ x \in [-2.1, -1.9]$ $\quad x^2 \in [3.61, 4.41]$

$y \in [2.9, 3.1]$ $\quad y - x^2 \leq 3.1 - 3.69 = -0.59 < 0$

$y - x^2 \neq 0$

f_x at $(-2,3)$ is $\dfrac{-4}{(-1)^2} = -4$

f_y at $(-2,3)$ is -1 $\quad u = \langle 1, 2\rangle$ $\quad \dfrac{\vec{u}}{|\vec{u}|} = \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$ $\quad \boxed{\frac{1}{\sqrt{5}} \cdot (-4) + \frac{2}{\sqrt{5}}(-1)}$

(b) $\vec{v} \,/\!/\, \langle f_x(-2,3), f_y(-2,3) \rangle$

$\vec{v} = \langle \frac{4}{\sqrt{17}}, \frac{1}{\sqrt{17}} \rangle$

✱ Didn't cover in either section:

7. Find the normal vector to the surface

$$x^2 + y^2 + z^2 = 65$$

at $(7, 0, 4)$. $\quad f(x,y,z) = x^2 + y^2 + z^2 = C$

$\langle f_x, f_y, f_z \rangle = \langle 2x, 2y, 2z \rangle$ gives normal vec

at $(7,0,4)$, $\langle 14, 0, 8 \rangle$; unit normal $\langle \frac{7}{\sqrt{65}}, 0, \frac{4}{\sqrt{65}} \rangle$

8. Find the normal vector to the surface

$$x^3 + yz^2 = 11$$

at $(3, -1, 4)$.

Let $f(x,y,z) = x^3 + yz^2$, take level surface at 11

$\langle f_x, f_y, f_z \rangle = \langle 3x^2, z^2, 2yz \rangle$ is the normal vector

at $(3,-1,4)$, $\langle 27, 16, -8 \rangle$;

unit normal $\left\langle \frac{27}{\sqrt{1049}}, \frac{16}{\sqrt{1049}}, \frac{-8}{\sqrt{1049}} \right\rangle$